RESEARCH ARTICLE

On Contra Semi Weakly $g^*$-Continuous Functions in Topological Spaces

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(Received: 24 August 2011, Accepted: 25 October 2011)

In this paper, we introduce the new class of weaker form of contra semi weakly $g^*$-continuous functions in topological spaces and study some of their properties.

Keywords: Contra semi weakly $g^*$-continuous maps.
AMS Subject Classification: 54C05, 54C08.

1. Introduction


Andrew and Whittlesey [15] introduced the notion of closure continuity. The $\delta$-continuity was introduced by Noiri [16]. In this paper we introduce new class of maps called contra semi weakly $g^*$-continuous maps which included the class of generalized continuous maps. Throughout this paper $(X, \tau)$ is a topological space $A$ is a subset of $(X, \tau)$. The closure of $A$ and interior of $A$ are denoted by $\text{Cl}(A)$ and $\text{Int}(A)$, respectively.

2. Preliminaries

Let us recall the following definitions which are useful in the sequel.

Definition 2.1 Let $(X, \tau)$ be a topological space. A subset $A$ of $X$ is said to be semi-open set [17] if $A \subseteq \text{Cl}(\text{Int}(A))$ and a semi closed set if $\text{Int}(\text{Cl}(A)) \subseteq A$.

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Definition 2.2 Let \((X, \tau)\) be a topological space. A sub set \(A\) of \(X\) is said to be preopen set \([10]\) if \(A \subseteq \text{Int}(\text{Cl}(A))\) and a preclosed set if \(\text{Cl}(\text{Int}(A)) \subseteq A\).

Definition 2.3 Let \((X, \tau)\) be a topological space. A sub set \(A\) of \(X\) is said to be \(\alpha\)-open set \([18]\) if \(A \subseteq \text{Int}(\text{Cl}(A))\) and a \(\alpha\)-closed set if \(\text{Cl}(\text{Int}(A)) \subseteq A\).

Definition 2.4 Let \((X, \tau)\) be a topological space. A sub set \(A\) of \(X\) is said to be semi preopen set \([19]\) or \(\beta\)-open if \(A \subseteq \text{Cl}(\text{Int}(A))\) and semi-preclosed set or \(\beta\)-closed \([1]\) if \(\text{Int}(\text{Cl}(A)) \subseteq A\).

Definition 2.5 Let \((X, \tau)\) be a topological space. A subset \(A\) of \(X\) is said to be regular open set \([20]\) if \(A = \text{Int}(\text{Cl}(A))\) and a regular closed set if \(\text{Cl}(\text{Int}(A)) = A\).

Definition 2.6 Let \((X, \tau)\) be a topological space. A sub set of \(A\) is said to be semi regular set \([21]\) if it both semi open and semi closed in \((X, \tau)\).

Definition 2.7 Let \((X, \tau)\) be a topological space. It is called a generalized closed set \([1]\) (briefly \(g\)-closed set) if \(\text{Cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \((X, \tau)\).

Definition 2.8 Let \((X, \tau)\) be a topological space. It is called a semi-generalized closed set \([22]\) (briefly \(gs\)-closed set) if \(s\text{Cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is semi open in \((X, \tau)\).

Definition 2.9 Let \((X, \tau)\) be a topological space. It is called a generalized semiclosed set \([23]\) (briefly \(gs\)-closed set) if \(\text{Cl}(\alpha \text{Cl}(A)) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \((X, \tau)\).

Definition 2.10 Let \((X, \tau)\) be a topological space. It is called a \(\alpha\)-generalized closed set \([24]\) (briefly \(\alpha g\)-closed set) if \(\alpha \text{Cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \((X, \tau)\).

Definition 2.11 Let \((X, \tau)\) be a topological space. It is called a generalized \(\alpha\)-closed set \([25]\) (briefly \(g\alpha\)-closed set) if \(\alpha \text{Cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\alpha\)-open in \((X, \tau)\).

Definition 2.12 Let \((X, \tau)\) be a topological space. It is called a semi-pre-closed set \([13]\) (briefly \(gsp\)-closed set) if \(s\text{Cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \((X, \tau)\).

Definition 2.13 Let \((X, \tau)\) be a topological space. It is called semi weakly \(g^*\)-closed set \([26]\) (briefly \(swg^*\)-closed set) if \(g\text{Cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is semi open.

Remark 2.14 The complement of respective closed sets are the corresponding open sets \([26]\) and vice versa.

Definition 2.15 A function \(f : (X, \tau) \rightarrow (Y, \sigma)\) is said to be semi-continuous \([17]\) if \(f^{-1}(V)\) is semi-open in \((X, \tau)\) for every open set \(V\) of \((Y, \sigma)\).

Definition 2.16 A function \(f : (X, \tau) \rightarrow (Y, \sigma)\) is said to be pre-continuous \([10]\) if \(f^{-1}(V)\) is pre-open in \((X, \tau)\) for every open set \(V\) of \((Y, \sigma)\).

Definition 2.17 A function \(f : (X, \tau) \rightarrow (Y, \sigma)\) is said to be \(\alpha\) -continuous \([27]\) if \(f^{-1}(V)\) is open in \((X, \tau)\) for every open set \(V\) of \((Y, \sigma)\).

Definition 2.18 A function \(f : (X, \tau) \rightarrow (Y, \sigma)\) is said to be \(\beta\) - continuous \([11]\) if \(f^{-1}(V)\) is semi preopen in \((X, \tau)\) for every open set \(V\) of \((Y, \sigma)\).

Definition 2.19 A function \(f : (X, \tau) \rightarrow (Y, \sigma)\) is said to be \(g\) -continuous \([9]\) if \(f^{-1}(V)\) is \(g\)-open in \((X, \tau)\) for every open set \(V\) of \((Y, \sigma)\).

Definition 2.20 A function \(f : (X, \tau) \rightarrow (Y, \sigma)\) is said to be \(sg\) -continuous if \(f^{-1}(V)\) is \(sg\)-open in \((X, \tau)\) for every open set \(V\) of \((Y, \sigma)\).

Definition 2.21 A function \(f : (X, \tau) \rightarrow (Y, \sigma)\) is said to be \(gs\)-continuous \([12]\) if \(f^{-1}(V)\) is \(gs\)-open in \((X, \tau)\) for every open set \(V\) of \((Y, \sigma)\).

Definition 2.22 A function \(f : (X, \tau) \rightarrow (Y, \sigma)\) is said to be \(ga\)- continuous \([25]\) if \(f^{-1}(V)\) is \(ga\)-open in \((X, \tau)\) for every open set \(V\) of \((Y, \sigma)\).
Definition 2.23 A function \( f : (X, \tau) \rightarrow (Y, \sigma) \) is said to be \( \alpha g \)-continuous \([28]\) if \( f^{-1}(V) \) is \( \alpha g \)-open in \((X, \tau)\) for every open set \( V \) of \((Y, \sigma)\).

Definition 2.24 A function \( f : (X, \tau) \rightarrow (Y, \sigma) \) is said to be \( g_{sp} \)-continuous \([13]\) if \( f^{-1}(V) \) is \( g_{sp} \)-open in \((X, \tau)\) for every open set \( V \) of \((Y, \sigma)\).

Definition 2.25 A function \( f : X \rightarrow Y \) is said to be \( rwg \)-continuous \([29]\) if the inverse image of every open set in \( Y \) is \( rwg \)-open in \( X \).

Definition 2.26 A function \( f : (X, \tau) \rightarrow (Y, \sigma) \) is called contra continuous \([2]\) if \( f^{-1}(V) \) is closed in \((X, \tau)\) for each open set \( V \) in \((Y, \sigma)\).

Definition 2.27 Let \( X \) and \( Y \) be topological spaces. A map \( f : X \rightarrow Y \) is said to be semi weakly \( g^* \)-continuous (\( swg^* \)-continuous) \([30]\) if the inverse image of every open set in \( Y \) is \( swg^* \)-open in \( X \).

Definition 2.28 A function \( f : (X, \tau) \rightarrow (Y, \sigma) \) is said to be contra semi -continuous \([3]\) if \( f^{-1}(V) \) is semi-closed in \((X, \tau)\) for every open set \( V \) of \((Y, \sigma)\).

Definition 2.29 A function \( f : (X, \tau) \rightarrow (Y, \sigma) \) is said to be contra precontinuous \([4]\) if \( f^{-1}(V) \) is preclosed in \((X, \tau)\) for every open set \( V \) of \((Y, \sigma)\).

Definition 2.30 A function \( f : (X, \tau) \rightarrow (Y, \sigma) \) is said to be contra \( \alpha \) -continuous \([5]\) if \( f^{-1}(V) \) is closed in \((X, \tau)\) for every open set \( V \) of \((Y, \sigma)\).

Definition 2.31 A function \( f : (X, \tau) \rightarrow (Y, \sigma) \) is said to be contra \( \beta \)- continuous \([31]\) if \( f^{-1}(V) \) is semi preclosed in \((X, \tau)\) for every open set \( V \) of \((Y, \sigma)\).

Definition 2.32 A function \( f : X \rightarrow Y \) is called almost continuous at \( x \in X \) if for every open set \( V \) in \( Y \) containing \( f(x) \), there is an open set \( U \) in \( X \) containing \( x \) such that \( f(V) \subset V^{\circ} \). If \( f \) is almost continuous at every point of \( X \) then it is called almost continuous.

3. Contra Semi Weakly \( g^* \)-Continuous Functions

In this section, we introduce contra semi weakly \( g^* \)-continuous functions and study some of their properties.

Definition 3.1 A function \( f : (X, \tau) \rightarrow (Y, \sigma) \) is called contra semi weakly \( g^* \)-continuous if \( f^{-1}(V) \) is \( swg^* \)-closed in \((X, \tau)\) for each open set \( V \) in \((Y, \sigma)\).

Theorem 3.2 If a function \( f : X \rightarrow Y \) is contra \( swg^* \)-continuous and \( Y \) is regular then \( f \) is \( swg^* \)-continuous.

Proof Let \( X \) be an arbitrary point of \( x \) and let \( V \) be an open set of \( Y \) containing \( f(x) \); since \( Y \) is regular there exist an open set \( W \) in \( Y \) containing \( f(x) \). Such that \( Cl(W) \subseteq V \). Since \( f \) is contra \( swg^* \)-continuous and there exists \( U \in SWG^*O(X, x) \) such that \( f(U) \subseteq Cl(W) \). Then \( f(U) \subseteq Cl(W) \subseteq V \).

Hence \( f \) is \( swg^* \)-continuous.

Theorem 3.3 If a function \( f : X \rightarrow Y \) is contra \( swg^* \)-continuous and \( X \) is \( swg^* \)-space, then \( f \) is contra continuous.

Proof Let \( V \) be a closed set in \( Y \). Since \( f \) is contra \( swg^* \)-continuous, \( f^{-1}(V) \) is \( swg^* \)-open in \( X \). Since \( X \) is \( swg^* \)-space \( f^{-1}(V) \) is open in \( X \). Hence \( f \) is contra continuous.

Corollary 3.1 If \( X \) is a \( swg^* \)-space then for a function \( f : X \rightarrow Y \) the following statements are equivalent:

1. \( f \) is contra continuous.
2. \( f \) is contra \( swg^* \)-continuous.

Proof Obvious.
Definition 3.4 Let \( A \) be a sub set of a space \((X, \tau)\):

1. The set \( \{ \text{F} \subseteq \text{X} \mid \text{A} \subseteq \text{F}, \text{F is swg*-closed} \} \) is called the swg*-closure of \( A \) and is denoted by \( \text{Cl}_{\text{swg}^*}(A) \).
2. The set \( \{ \text{F} \subseteq \text{X} \mid \text{F is swg*-open} \} \) is called swg*-interior of \( A \) and is denoted by \( \text{Int}_{\text{swg}^*}(A) \).

Lemma 3.5 The following properties hold for sub sets \( A, B \) of a space \( X \).

1. \( x \in \text{ker}(A) \), if and only if \( A \cap f \neq \phi \) for any \( f \in C(X, x) \).
2. \( A \subseteq \text{ker}(A) \) and \( A = \text{ker}(A) \) if \( A \) is open in \( X \).
3. If \( A \subseteq B \), then \( \text{ker}(A) \subseteq \text{ker}(B) \).

Theorem 3.6 Let \( A \) be a subset of \((X, \tau)\).

1. If \( A \) is swg*-closed then \( g \text{Cl}(A) - A \) does not contain any non-empty \#gs-closed set.
2. If \( A \) is swg*-closed and \( A \subseteq B \subseteq g \text{Cl}(A) \), then \( B \) is swg*-closed.

Proof (1) Suppose that \( A \) is swg*-closed and let \( F \) be a non-empty \#gs-closed set with \( F \subseteq g \text{Cl}(A) - A \). Then \( A \subseteq X - F \) and so \( g \text{Cl}(A) \subseteq X - F \). Hence \( F \subseteq X - g \text{Cl}(A) \), a contradiction.

(2) Let \( U \) be a \#gs-open set of \((X, \tau)\) such that \( B \subseteq U \). Then \( A \subseteq U \) since \( A \) is swg*-closed \( g \text{Cl}(A) \subseteq U \). Since \( A \) is swg*-closed \( g \text{Cl}(A) \subseteq U \). Now \( g \text{Cl}(B) \subseteq g \text{Cl}(g \text{Cl}(A)) \subseteq U \). Therefore \( B \) is also a swg*-closed set of \((X, \tau)\).

Theorem 3.7 For a function \( f : (X, \tau) \to (Y, \sigma) \) the following conditions are equivalent:

1. \( f \) is contra swg*-continuous;
2. For every closed subset \( F \) of \( Y \), \( f^{-1}(F) \subseteq SWG \ast O(X) \);
3. For each \( x \in X \) and each \( F \in C(Y, f(x)) \), there exists \( U \in SWG \ast O(X, x) \) such that \( f(U) \subseteq F \);
4. \( f(Cl_{\text{swg}^*}(A)) \subseteq \text{ker}(f(A)) \) for every sub set \( A \) of \( X \);
5. \( Cl_{\text{swg}^*}(f^{-1}(B)) \subseteq f^{-1}(\text{ker}(B)) \) for every sub set \( B \) of \( Y \).

Proof The implication (1) \(\Rightarrow\) (2) and (2) \(\Rightarrow\) (3) are obvious.

(3)\(\Rightarrow\)(2): Let \( F \) be any closed set of \( Y \) and \( X \in f^{-1}(F) \). Then \( f(x) \in F \) and there exists \( U_x \in swg \ast O(X, x) \) such that \( f(U_x) \subseteq F \). Therefore we obtain \( f^{-1}(F) = \cup\{U_x : X \in f^{-1}(F)\} \) and \( f^{-1}(F) \) is swg*-open, by Theorem 3.6.

(3)\(\Rightarrow\)(4): Let \( A \) be any subset of \( X \). Suppose that \( Y \notin \text{ker}(f(A)) \). Then by Lemma 3.5, there exists \( F \in C(Y, f(x)) \) such that \( f(A) \cap F = \phi \). Thus we have \( A \cap f^{-1}(F) = \phi \) and since \( f^{-1}(F) \) is swg*-open then we have \( Cl_{\text{swg}^*}(A) \cap f^{-1}(F) = \phi \). Therefore, we obtain \( f(Cl_{\text{swg}^*}(A) \cap F = \phi \) and \( Y \notin f(Cl_{\text{swg}^*}(A)) \). This implies that \( f(Cl_{\text{swg}^*}(A)) \subseteq \text{ker}(f(A)) \).

(4)\(\Rightarrow\)(5): Let \( B \) be any subset of \( Y \) by (4) and Lemma 3.5, we have \( f(Cl_{\text{swg}^*}(f^{-1}(B)) \subseteq \text{ker}(f(f^{-1}(B))) \subseteq \text{ker}B \). Thus \( Cl_{\text{swg}^*}(f^{-1}(B)) \subseteq f^{-1}(\text{ker}(B)) \).

(5)\(\Rightarrow\)(1): Let \( V \) be any open set of \( Y \). Then by Lemma 3.5, we have \( Cl_{\text{swg}^*}(f^{-1}(V)) \subseteq f^{-1}(\text{ker}(V)) = f^{-1}(V) \). Thus shows that \( f^{-1}(V) \) is swg*-closed in \( X \).

Theorem 3.8 Let \( f : X \to Y \) be a function then the following are equivalent:

1. The function \( f \) is swg*-continuous;
2. For each point \( x \in X \) and each open set \( V \) of \( Y \) with \( f(x) \in V \), there exists a swg*-open set \( U \) of \( X \) such that \( x \in U, f(U) \subseteq V \).

Proof (1) \(\Rightarrow\)(2): Let \( f(x) \in V \). Then \( x \in f^{-1}(V) \in SWG \ast O(X) \), since \( f \) is swg*-continuous. Let \( U = f^{-1}(V) \), then \( x \in X \) and \( f(U) \subseteq V \).

(2)\(\Rightarrow\)(1): Let \( V \) be an open set of \( Y \) and let \( x \in f^{-1}(V) \). Then \( f(x) \in V \). Then \( f(x) \in V \) and thus there exists on \( swg^*\)-open set \( U_x \) of \( X \) such that \( x \in U_x \) and \( f(U) \subseteq V \). Now \( x \in U_x \subseteq f^{-1}(V) \) and \( f^{-1}(V) = U_x \). Then \( f^{-1}(V) \) is swg*-open in \( X \). Therefore \( f \) is swg*-continuous.
Definition 3.9 A function $f : X \to Y$ is called almost $swg*$-continuous if for each $x \in X$ and each open set $V$ of $Y$ containing $f(x)$, there exists $U \in SWG \ast O(X,x)$ such that $f(V) \subseteq Int_{swg^*}(Cl(V))$.

Theorem 3.10 A function $f : X \to Y$ is almost $swg*$-continuous if and only if for each $x \in X$ and each regular open set $V$ of $Y$ containing $f(x)$, there exists $U \in SWG \ast O(X,x)$ such that $f(U) \subseteq V$.

Proof Let $V$ be a regular open set of $Y$ containing $f(x)$ for each $x \in X$. This implies that $V$ is an open set of $X$ containing $f(x)$ for each $x \in X$. Since $f$ is almost $swg*$-continuous, there exist $U \in SWG \ast O(X,x)$ such that $f(U) \subseteq V$. Conversely, if for each $x \in X$ and each regular open set $V$ of $Y$ containing $f(x)$, there exists $U \in SWG \ast O(X,x)$ such that $f(U) \subseteq V$. This implies $V$ is an open set of $Y$ containing $f(x)$, there exists $U \in SWG \ast O(X,x)$ such that $f(U) \subseteq V$. Therefore $f$ is almost $swg*$-continuous.

Definition 3.11 A function $f : X \to Y$ is said to be pre $swg*$-open if the image of each $swg*$-open set is $swg*$-open.

Theorem 3.12 If a function $f : X \to Y$ is a pre $swg*$-open and contra $swg*$-continuous then $f$ is almost $swg*$-continuous.

Proof Let $x$ be any arbitrary point of $X$ and $V$ be an open set containing $f(x)$. Since $f$ is contra $swg*$-continuous then by Theorem 3.7 (3), there exists $V \in SWG \ast O(X,x)$, such that $f(V) \subseteq Cl(V)$. Since $f$ is pre $swg*$-open, $f(U)$ is $swg*$-open in $Y$. Therefore, $f(U) = Int_{swg^*}f(U) \subseteq Int_{swg^*}(Cl(f(U))) \subseteq Int_{swg^*}(Cl(V))$. This shows that $f$ is almost $swg*$-continuous.

Definition 3.13 The $swg*$-frontier of $A$ of a space $(X,\tau)$, denoted by $Fr_{swg^*}(A)$ is defined by $Fr_{swg^*}(A) = Cl_{swg^*}(A) \cap Cl_{swg^*}(X - A)$.

Theorem 3.14 If $K = \{x : X, V \cap U \neq \phi, U \subseteq X\}$ for every $swg*$-open set $V$ containing $x$, then $Cl_{swg^*}(U) = K$.

Proof Let $x \in K \iff V \cap U \neq \phi, x \in V, V$ is a $swg*$-open set, $\iff x \in V$ or every $swg*$-open set containing $x$ contains a point of $U$ other then $X$, $\iff x \in Cl_{swg^*}(U)$.

Theorem 3.15 The set of all points $x$ of $X$ at which $f : X \to Y$ is not contra $swg*$-continuous is identical with the union of the $swg*$-frontier of the inverse image of closed sets of $Y$ containing $f(x)$.

Proof Suppose $f$ is not contra $swg*$-continuous at $x \in X$. There exists $F \in C(Y,f(x))$, such that $f(U) \cap (Y - F) \neq \phi$ for every $U \in SWG \ast O(X,x)$. This implies that $U \cap f^{-1}(Y - F) \neq \phi$. Therefore we have $x \in Cl_{swg^*}(f^{-1}(Y - F)) = Cl_{swg^*}(X - f^{-1}(F))$. However $x \in Cl_{swg^*}(f^{-1}(F)) \cap Cl_{swg^*}(f^{-1}(Y - F))$. Therefore we obtain $x \in Fr_{swg^*}(f^{-1}(F))$. Suppose that $x \in Fr_{swg^*}(f^{-1}(F))$ for some $F \in C(Y,f(x))$ and $f$ is contra $swg*$-continuous at $x$, then there exists $U \in SWG \ast O(X,x)$. Such that $f(U) \subseteq F$. Therefore, we have $x \in U \subseteq f^{-1}(F)$ and hence $x \in Int_{swg^*}(f^{-1}(F)) \subseteq X - Fr_{swg^*}(f^{-1}(F))$. This is a contradiction. This means that $f$ is not contra $swg*$-continuous.

Theorem 3.16 Let $\{X_{\lambda} : \lambda \in \Lambda\}$ be any family of topological space. If $f : X \to \pi X_{\lambda}$ is a contra $swg*$-continuous function. Then $P_{X_{\lambda}}f : X \to X_{\lambda}$ is a contra $swg*$-continuous for each $\lambda \in \Lambda$, where $P_{\lambda}$ is the projection of $\pi X_{\lambda}$ onto $X_{\lambda}$.

Proof We shall consider a fixed $\lambda \in \Lambda$. Suppose $U_{\lambda}$ is an arbitrary open set in $X_{\lambda}$. Then $P_{X_{\lambda}}^{-1}(U_{\lambda})$ is open in $\pi X_{\lambda}$. Since $f$ is contra $swg*$-continuous, we have by definition $f^{-1}(P_{\lambda}^{-1}(U_{\lambda})) = (P_{X_{\lambda}}f)^{-1}(U_{\lambda})$ is $swg*$-closed in $X$. Therefore $P_{X_{\lambda}}f$ is contra $swg*$-continuous.

Theorem 3.17 If $f : X \to Y$ be surjective $swg*$-irresolute and pre $swg*$-open and $g : Y \to Z$ be any function. Then $g \circ f : X \to Z$ is contra $swg*$-continuous if and only if $g$ is contra $swg*$-continuous.

Proof The if part is obvious. To prove the only if part, let $g \circ f : X \to Z$ is contra $swg*$-continuous and let $F$ be a closed subset of $Z$. Then $(g \circ f)^{-1}(F)$ is a $swg*$-open of $X$. That is $f^{-1}(g^{-1}(F))$ is
an $swg$-open subset of $X$, since $f$ is pre $swg$-open $f(f^{-1}(g^{-1}(F)))$ is $swg$-open subset of $Y$. So $g^{-1}(F)$ is an $swg$-open in $Y$. Hence $g$ is contra $swg$-continuous.

For function $f : X \to Y$, the subset $\{(x, f(x))| x \in X\} \subseteq X \times Y$ is called the graph of $f$ and is denoted by $Gr(f)$. 

Definition 3.18 The graph $Gr(f)$ of a function $f : X \to Y$ is said to be contra $swg$-closed if for each $(x, y) \in (X, Y) - Gr(f)$, there exists $U \in SWG \ast O(X, x)$ and $V \in C(Y, y)$ such that $(U \times V) \cap (U \times V) = \phi$, symbolically we say $f$ is $CSwg$-closed in the product space $X \times Y$.

Lemma 3.19 Let $Gr(f)$ be the graph of $f$, for any subset $A \subseteq X$ and $B \subseteq Y$ we have $f(A) \cap B = \phi$ if and only if $(A \times B) \cap G(f) = \phi$.

Lemma 3.20 The Graph $Gr(f)$ of a function $f : X \to Y$ is $CSwg$-closed in $X \times Y$ if and only if for each $(x, y) \in (X, Y) - Gr(f)$, there exists $U \in SWG \ast O(X, x)$ and $V \in (Y, y)$ such that $f(U) \cap V = \phi$.

Theorem 3.21 If $f : X \to Y$ is contra $swg$-continuous and $Y$ is Urysohn, then $f$ is $CSwg$-closed in the product space $X \times Y$.

Proof Let $(x, y) \in (X \times Y) - Gr(f)$. Then $y \neq f(x)$ and there exists open sets $H_1$, $H_2$ such that $f(x) \in H_1$, $y \in H_2$ and $Cl(H_1) \cap Cl(H_2) = \phi$. From Hypothesis, there exists $V \in SWG \ast O(X, x)$ such that $f(V) \subseteq Cl(H_1)$. Therefore, we obtain $f(V) \cap Cl(H_2) = \phi$. This shows that $f$ is $CSwg$-closed.

Theorem 3.22 If $f : X \to Y$ and $g : X \to Y$ are contra $swg$-continuous and $Y$ is Urysohn, then $K = \{(x \in X, f(x) = g(x)\}$ is $swg$-closed in $X$.

Proof Let $x \in X - K$. Then $f(x) \neq g(x)$ since $Y$ is Urysohn, there exists open sets $U$ and $V$ such that $f(x) \in U$, $g(x) \in V$ and $Cl(U) \cap Cl(V) = \phi$. Since $f$ and $g$ are contra $swg$-continuous $f^{-1}(Cl(U)) \in SWG \ast O(X)$ and $g^{-1}(Cl(V)) \in SWG \ast O(X)$. Let $A = f^{-1}(Cl(U))$ and $B = g^{-1}(Cl(V))$, then $A$ and $B$ contains $X$, set $C = A \cap B$, then $C$ is $swg$-open in $X$. Hence $f(C) \cap g(C) = \phi$ and $x \notin Cl_{swg}(K)$. Thus $K$ is $swg$-closed in $X$.

Theorem 3.23 Let $f : X \to Y$ be a function and let $g : X \to X \times Y$ be the graph function of $f$, defined by $g(x) = (x, f(x))$ for every $x \in X$. If $g$ is contra $swg$-continuous, then $f$ is contra $swg$-continuous.

Proof Let $U$ be an open set in $Y$ then $X \times U$ is an open set in $X \times Y$. Since $g$ is contra $swg$-continuous, it follows that $f^{-1}(U) = g^{-1}(X \times U)$ is an $swg$-closed in $X$. Thus $f$ is contra $swg$-continuous.

Theorem 3.24 If $f : X \to Y$ is $swg$-continuous and $Y$ is $T_1$, there $f$ is $CSwg$-closed in $X \times Y$.

Proof Let $(x, y) \in (X \times Y) - Gr(f)$. Then $f(x) \neq y$ and there exists an open set $V$ of $Y$ such that $f(x) \in V$ and $y \notin V$. Since $f$ is $swg$-continuous there exists $U \in SWG \ast O(X, x)$. Such that $f(U) \subseteq V$. Therefore, we have $f(U) \cap (Y - V) = \phi$ and $Y - U \in C(Y, y)$. This shows that $f$ is $CSwg$-closed in $X \times Y$.

Definition 3.25

1. A space $X$ is said to be $swg$-$T_1$ if for each pair of distinct points $x$ and $y$ in $X$, there exists $swg$-open sets $U$ and $V$ containing $x$ and $y$, respectively, such that $y \notin U$ and $x \notin V$.

2. A space $X$ is said to be $swg$-$T_2$ if for each pair of distinct points $x$ and $y$ in $X$, there exists $swg$-open sets $U$ and $V$ containing $x$ and $y$ respectively such that $U \cap V = \phi$.

Theorem 3.26 Let $X$ be a topological and for each pair of distinct points $x$ and $y$ in $X$, there exists a map $f$ of $X$ into a urysohn topological space $Y$ such that $f(x) \neq f(y)$ and $f$ is contra $swg$-continuous at $x$ and $y$, then $X$ is $swg$-$T_2$.

Proof Let $x$ and $y$ be any two distinct points in $X$. Then there exists and urysohn space $Y$ and a function $f : X \to Y$ such that $f(x) \neq f(y)$ and $f$ is contra $swg$-continuous at $x$ and $y$. Let $a = f(x)$ and $b = f(y)$. Then $a \neq b$, since $Y$ is urysohn, there exists open sets $V$ and $W$ containing $a$ and $b$, respectively such that $Cl(Y) \cap Cl(W) = \phi$. Since $f$ is contra $swg$-continuous at $x$ and $y$, there exists
swg*-open sets $A$ and $B$ containing $a$ and $b$, respectively, such that $f(A) \subseteq Cl(V)$ and $f(B) \subseteq Cl(W)$. Then $f(A) \cap f(B) = \phi$. So $A \cap B = \phi$. Hence $X$ is swg*-T$_2$.

Corollary 3.2 Let $f : X \to Y$ be contra swg*-continuous injection. If $Y$ is an urysohn space, then $X$ is swg*-T$_2$.

Definition 3.27 A space $X$ is said to be weakly hausdorff, if each element of $X$ is an intersection of regular closed sets.

Theorem 3.28 If $f : X \to Y$ is a contra swg*-continuous injection and $Y$ is weakly hausdorff, then $X$ is swg*-T$_1$.

Proof Suppose that $Y$ is weakly hausdorff. For any distinct points $x_1$ and $x_2$ in $X$, there exists regular closed sets $U$ and $V$ in $Y$. such that $f(x_1) \in U$, $f(x_2) \notin U$, $f(x_1) \notin V$ and $f(x_2) \in V$. Since $f$ is contra swg*-continuous $f^{-1}(U)$ and $f^{-1}(V)$ are swg*-open, subsets of $X$, such that $x_1 \in f^{-1}(U)$, $x_2 \notin f^{-1}(U)$, $x_1 \notin f^{-1}(V)$ and $x_2 \in f^{-1}(V)$. This shown that $X$ is swg*-T$_1$.

Theorem 3.29 Let $f : X \to Y$ have a $CSwg*$-closed graph. If $f$ is injective, then $X$ is swg*-T$_1$.

Proof Let $x_1$ and $x_2$ be any two distinct points of $X$. Then we have $(x_1, f(x_2)) \in (X \times Y) - G(f)$. There exist a swg*-open set $U$ in $X$. Containing $x_1$ and $F \subseteq C(Y, f(x_2))$ such that $f(U) \cap F = \phi$. Hence $U \cap f^{-1}(F) = \phi$. Therefore we have $x_2 \notin U$. This implies that $x$ is swg*-T$_1$.

Definition 3.30 A topological space is said to be ultra hausdorff, if for each pair of distinct points $x$ and $y$ in $X$, there exists clopen sets $A$ and $B$ containing $x$ and $y$ respectively such that $A \cap B = \phi$.

Theorem 3.31 Let $f : X \to Y$ be a contra swg*- continuous injection. If $Y$ is ultra hausdorff space, then $X$ is swg*-T$_2$.

Proof Let $x_1$ and $x_2$ be any two distinct points of $X$, then $f(x_1) \neq f(x_2)$ and there exist clopen sets $U$ and $V$ containing $f(x_1)$ and $f(x_2)$, respectively such that $U \cap V = \phi$. Since $f$ is contra swg*-continuous, then $f^{-1}(U) \in SWG*O(X)$ and $f^{-1}(V) \in SWG*O(X)$ such that $f^{-1}(U) \cap f^{-1}(V) = \phi$. Hence $X$ is swg*-T$_2$.

Remark 3.32 The following examples show that contra swg*-continuous function and contra $\alpha$-continuous function are independent.

Example 3.33 Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \phi, \{b\}\}$ and $\sigma = \{Y, \phi, \{a, c\}, \{a, c\}\}$. Consider $f : X \to Y$ defined as $f(a) = a$, $f(b) = b = f(c)$. This function $f$ is contra swg*-continuous but not contra $\alpha$-continuous. Since the pre-image of the open set $\{a\}$ in $Y$ is $\{a\}$ is not $\alpha$-open in $X$.

Example 3.34 Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{c\}\}$. Consider $f : X \to Y$ defined as $f(a) = b = f(b)$, $f(c) = a$. This function $f$ is contra $\alpha$-continuous but not contra swgs-continuous. Since for the pre-image of the open set $\{a, b\}$ in $Y$ is $\{a, b\}$ is not swgs-open in $X$.

Remark 3.35 The following examples show that contra swgs-continuous function and contra semi continuous function are independent.

Example 3.36 Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}\}$ and $\sigma = \{Y, \phi, \{a\}\}$ and $f$ be the identity map. This $f$ is contra swgs-continuous but not contra semi continuous as the inverse image of this open set $\{b, c\}$ in $Y$ is $\{b, c\}$ in $X$ is not semiopen.

Example 3.37 Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \phi, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{a, b\}\}$. Consider $f : X \to Y$ defined as $f(a) = a$, $f(b) = f(c) = c$. This function $f$ is contra semi continuous but not contra swgs-continuous since the pre image of the open set $\{c\}$ in $Y$ is $\{c\}$ is not swgs-open in $X$.

Remark 3.38 The following examples show that contra swgs-continuous function and contra pre continuous function are independent.
Example 3.39 Let \( X = Y = \{a, b, c\} \) with \( \tau = \{X, \phi, \{a\}, \{a, c\}\} \) and \( \sigma = \{Y, \phi, \{a, b\}\} \). Consider \( f : X \to Y \) defined as \( f(a) = b, f(b) = a, f(c) = c \). The function \( f \) is contra swg*-continuous but not contra precontinuous as inverse image of this open set \( \{c\} \) in \( Y \) is \( \{c\} \) in \( X \) is not preopen.

Example 3.40 Let \( X = Y = \{a, b, c\} \) with \( \tau = \{X, \phi, \{a\}, \{a, b\}\} \) and \( \sigma = \{Y, \phi, \{b, c\}\} \). Consider \( f : X \to Y \) defined as \( f(a) = a, f(b) = a, f(c) = c \). This function \( f \) is contra precontinuous but not contra swg*-continuous as inverse image of this open set \( \{a, c\} \) in \( Y \) is \( \{a, c\} \) in \( X \) is not swg*-open.

Example 3.42 Let \( X = Y = \{a, b, c\} \) with \( \tau = \{X, \phi, \{c\}\} \) and \( \sigma = \{Y, \phi, \{c\}\} \) and \( f \) be the identity map. This function \( f \) is contra swg*-continuous but not contra \( \beta \)-continuous as the inverse image of this open set \( \{a, b\} \) in \( Y \) is \( \{a, b\} \) in \( X \) is not \( \beta \)-open.

Example 3.43 Let \( X = Y = \{a, b, c\} \) with \( \tau = \{X, \phi, \{a, c\}\} \) and \( \sigma = \{Y, \phi, \{c\}\} \). Consider \( f : X \to Y \) defined as \( f(a) = b, f(b) = a, f(c) = c \). This function \( f \) is contra \( \beta \)-continuous but not contra swg*-continuous as the inverse image of this open set \( \{a, b\} \) in \( Y \) is \( \{a, b\} \) in \( X \) is not swg*-open.

Remark 3.41 The following examples show that contra swg*-continuous function and contra \( \beta \)-continuous function are independent.

Remark 3.42 From the above results we get the following diagram:

```
Contra semicontinuous
   /   /
/     /
Contra \( \alpha \)-continuous \( \xrightarrow{\sim} \) Contra swg*-continuous \( \xleftarrow{\sim} \) Contra \( \beta \)-continuous
   /   /
/     /
Contra precontinuous
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References

REFERENCES


