α-g-bicontinuous, α-g-bi-irresolute, α-g-compact and α-g-stable in complemented ditopological texture spaces

Hariwan Z. Ibrahim

Abstract
In this paper, the author introduce and study new notions of continuity, compactness and stability in ditopological texture spaces based on the notions of α-g-open and α-g-closed sets and some of their characterizations are obtained.

Keywords: Texture, difunction, α-g-bicontinuity, α-g-bi-irresolute, α-g-compactness, α-g-stability.

2010 MSC: 54A40.

1. Introduction and Preliminaries
Textures and ditopological texture spaces were first introduced by L. M. Brown as a point-based setting for the study fuzzy topology. The study of compactness and stability in ditopological texture spaces was begun in [1]. In this paper, we introduce and study the concepts of α-g-bicontinuity, α-g-bi-irresolute, α-g-compactness and α-g-stability in ditopological textures spaces.

The following are some basic definitions of textures we will need later on.

Texture space: Let S be a set. Then ϕ ⊆ P(S) is called a texturing of S, and S is said to be textured by ϕ if

(1) (ϕ, ⊆) is a complete lattice containing S and φ and for any index set I and A_i ∈ ϕ, i ∈ I, the meet \bigwedge_{i \in I} A_i and the join \bigvee_{i \in I} A_i in ϕ are related with the intersection and union in P(S) by the equalities

\[ \bigwedge_{i \in I} A_i = \bigcap_{i \in I} A_i \]

for all I, while

\[ \bigvee_{i \in I} A_i = \bigcup_{i \in I} A_i \]

for all finite I.
(2) \( \varphi \) is completely distributive.

(3) \( \varphi \) separates the points of \( S \). That is, given \( s_1 \neq s_2 \) in \( S \) we have \( L \in \varphi \) with \( s_1 \in L, s_2 \notin L \), or \( L \in \varphi \) with \( s_2 \in L, s_1 \notin L \).

If \( S \) is textured by \( \varphi \) then \((S,\varphi)\) is called a texture space, or simply a texture.

**Complementation:** \( \square \) A mapping \( \sigma : \varphi \rightarrow \varphi \) satisfying \( \sigma(\sigma(A)) = A \), \( \forall A \in \varphi \) and \( A \subseteq B \Rightarrow \sigma(B) \subseteq \sigma(A) \), \( \forall A,B \in \varphi \) is called a complementation on \((S,\varphi)\) and \((S,\varphi,\sigma)\) is then said to be a complemented texture.

For a texture \((S,\varphi)\), most properties are conveniently defined in terms of the \( p \)-sets

\[ P_s = \bigcap \{ A \in \varphi : s \in A \} \]

and the \( q \)-sets,

\[ Q_s = \bigvee \{ A \in \varphi : s \notin A \}. \]

**Ditopology:** \( \square \) A dichotomous topology on a texture \((S,\varphi)\), or ditopology for short, is a pair \((\tau,k)\) of subsets of \( \varphi \), where the set of open sets \( \tau \) satisfies:

1. \( S,\phi \in \tau \),
2. \( G_1,G_2 \in \tau \Rightarrow G_1 \cap G_2 \in \tau \), and
3. \( G_i \in \tau, i \in I \Rightarrow \bigvee_i G_i \in \tau \),

and the set of closed sets \( k \) satisfies:

1. \( S,\phi \in k \),
2. \( K_1,K_2 \in k \Rightarrow K_1 \cup K_2 \in k \), and
3. \( K_i \in k, i \in I \Rightarrow \bigcap_i K_i \in k \).

Hence a ditopology is essentially a “topology” for which there is no a priori relation between the open and closed sets.

For \( A \in \varphi \) we define the closure \([A]\) and the interior \(]A[\) of \( A \) under \((\tau,k)\) by the equalities

\[ [A] = \bigcap \{ K \in \varphi : A \subseteq K \} \quad \text{and} \quad ]A[= \bigvee \{ G \in \tau : G \subseteq A \}. \]

We refer to \( \tau \) as the topology and \( k \) as the cotopology of \((\tau,k)\).

If \((\tau,k)\) is a ditopology on a complemented texture \((S,\varphi,\sigma)\), then we say that \((\tau,k)\) is complemented if the equality \( k = \sigma[\tau] \) is satisfied. In this study, a complemented ditopological texture space is denoted by \((S,\varphi,\tau,k,\sigma)\). In this case we have \( \sigma([A]) = ]A[ \) and \( \sigma([A]) = [\sigma(A)] \).

We denote by \( O(S,\varphi,\tau,k) \), or when there can be no confusion by \( O(S) \), the set of open sets in \( \varphi \). Likewise, \( C(S,\varphi,\tau,k) \), \( C(S) \) will denote the set of closed sets.

Let \((S_1,\varphi_1)\) and \((S_2,\varphi_2)\) be textures. In the following definition we consider the product texture \( P(S_1) \otimes \varphi_2 \), and denote by \( P_{s,t}, Q_{s,t} \) respectively the \( p \)-sets and \( q \)-sets for the product texture \((S_1 \times S_2, P(S_1) \otimes \varphi_2)\).

**Dirrelation:** \[3\] Let \((S_1,\varphi_1)\) and \((S_2,\varphi_2)\) be textures. Then:

1. \( r \in P(S_1) \otimes \varphi_2 \) is called a relation from \((S_1,\varphi_1)\) to \((S_2,\varphi_2)\) if it satisfies
   
   R1: \( r \not\subseteq Q_{s,t}, P_s \not\subseteq Q_s \Rightarrow r \not\subseteq Q_{s,t}' \).
   
   R2: \( r \not\subseteq Q_{s,t} \Rightarrow \exists s' \in S_1 \) such that \( P_s \not\subseteq Q_{s'} \) and \( r \not\subseteq Q_{s',t} \).

2. \( R \in P(S_1) \otimes \varphi_2 \) is called a corelation from \((S_1,\varphi_1)\) to \((S_2,\varphi_2)\) if it satisfies:
   
   CR1: \( P_{s,t} \not\subseteq R, P_s \not\subseteq Q_s \Rightarrow P_{s',t} \not\subseteq R \).
   
   CR2: \( P_{s,t} \not\subseteq R \Rightarrow \exists s' \in S_1 \) such that \( P_s \not\subseteq Q_s \) and \( P_{s',t} \not\subseteq R \).
A pair \((r, R)\), where \(r\) is a relation and \(R\) a relation from \((S_1, \varphi_1)\) to \((S_2, \varphi_2)\) is called a direlation from \((S_1, \varphi_1)\) to \((S_2, \varphi_2)\).

One of the most useful notions of (ditopological) texture spaces is that of difunction. A difunction is a special type of direlation.

**Difunctions:** [3] Let \((f, F)\) be a direlation from \((S_1, \varphi_1)\) to \((S_2, \varphi_2)\). Then \((f, F)\) is called a difunction from \((S_1, \varphi_1)\) to \((S_2, \varphi_2)\) if it satisfies the following two conditions:

**DF1:** For \(s, s' \in S_1\), \(P_s \subseteq Q_{s'} \Rightarrow \exists t \in S_2\) such that \(f \not\subseteq \overline{Q}_{s,t} \text{ and } P_{s'} \not\subseteq F\).

**DF2:** For \(t, t' \in S_2\) and \(s \in S_1\), \(f \not\subseteq \overline{Q}_{s,t} \text{ and } P_{s,t'} \not\subseteq F \Rightarrow P_t \not\subseteq Q_t\).

**Image and Inverse Image:** [3] Let \((f, F) : (S_1, \varphi_1) \rightarrow (S_2, \varphi_2)\) be a difunction.

(1) For \(A \in \varphi_1\), the image \(f^{-A}\) and the co-image \(F^{-A}\) are defined by

\[
f^{-A} = \bigcap \{Q_t : \forall s, f \not\subseteq \overline{Q}_{s,t} \Rightarrow A \subseteq Q_s\},
\]

\[
F^{-A} = \bigvee \{P_t : \forall s, P_{s,t} \not\subseteq F \Rightarrow P_s \subseteq A\}.
\]

(2) For \(B \in \varphi_2\), the inverse image \(f^{-B}\) and the inverse co-image \(F^{-B}\) are defined by:

\[
f^{-B} = \bigvee \{P_s : \forall t, f \not\subseteq \overline{Q}_{s,t} \Rightarrow P_t \subseteq B\},
\]

\[
F^{-B} = \bigcap \{Q_s : \forall t, P_{s,t} \not\subseteq F \Rightarrow B \subseteq Q_t\}.
\]

For a difunction, the inverse image and the inverse co-image are equal, but the image and co-image are usually not.

**Bicontinuity:** [4] The difunction \((f, F) : (S_1, \varphi_1, \tau_1, k_1) \rightarrow (S_2, \varphi_2, \tau_2, k_2)\) is called continuous if \(B \in \tau_2 \Rightarrow F^{-B} \in \tau_1\), cocontinuous if \(B \in k_2 \Rightarrow f^{-B} \in k_1\), and bicontinuous if it is both continuous and cocontinuous.

**Surjective difunction:** [3] Let \((f, F) : (S_1, \varphi_1) \rightarrow (S_2, \varphi_2)\) be a difunction. Then \((f, F)\) is called surjective if it satisfies the condition:

**SUR.** For \(t, t' \in S_2\), \(P_t \not\subseteq Q_{t'} \Rightarrow \exists s \in S_1\) with \(f \not\subseteq \overline{Q}_{s,t'}\) and \(P_{s,t} \not\subseteq F\).

If \((f, F)\) is surjective then \(F^{-B} = f^{-B} = (f^{-B})\) for all \(B \in \varphi_2\) [3 Corollary 2.33].

**Definition 1.1.** [3] Let \((f, F)\) be a difunction between the complemented textures \((S_1, \varphi_1, \sigma_1)\) and \((S_2, \varphi_2, \sigma_2)\). The complement \((f, F)' = (f', F')\) of the difunction \((f, F)\) is a difunction, where \(f' = \bigcap \{Q_{s,t} : \exists u, v\) with \(f \not\subseteq \overline{Q}_{s,u,v} \cap Q_s = Q_{s,u} \cup P_v \not\subseteq \sigma_2(P_t)\}\) and \(F' = \bigcup \{P_{s,t} : \exists u, v\) with \(P_{u,v} \not\subseteq F, P_u \not\subseteq Q_s = \sigma_1(Q_t) \cup Q_v\}\). If \((f, F) = (f', F')\) then the difunction \((f, F)\) is called complemented.

**Definition 1.2.** [5] Let \((S, \varphi, \tau, k)\) be a ditopological texture space. A set \(A \in \varphi\) is called \(\alpha\)-open (\(\alpha\)-closed) if \(A \subseteq \overline{\overline{A}} \subseteq \overline{\overline{A}} \subseteq A\).

We denote by \(O_\alpha(S, \varphi, \tau, k)\), or when there can be no confusion by \(O_\alpha(S)\), the set of \(\alpha\)-open sets in \(\varphi\). Likewise, \(C_\alpha(S, \varphi, \tau, k)\), or \(C_\alpha(S)\) will denote the set of \(\alpha\)-closed sets.

**Definition 1.3.** [7] Let \((S, \varphi, \tau, k)\) be a ditopological texture space. A subset \(A\) of a texture \(\varphi\) is said to be generalized closed (g-closed for short) if \(A \subseteq G \in \tau\) then \([A] \subseteq G\).

**Definition 1.4.** [7] Let \((S, \varphi, \tau, k)\) be a complemented ditopological texture space. A subset \(A\) of a texture \(\varphi\) is said to be generalized open (g-open for short) if \(\sigma(A)\) is g-closed.

We denote by \(gc(S, \varphi, \tau, k)\), or when there can be no confusion by \(gc(S)\), the set of g-closed sets in \(\varphi\). Likewise, \(go(S, \varphi, \tau, k, \sigma)\), or \(go(S)\) will denote the set of g-open sets.
Definition 1.5. [6] Let \((S, \varphi, \tau, k)\) be a ditopological texture space. A subset \(A\) of a texture \(\varphi\) is said to be \(\alpha\)-g-closed if \(A \subseteq G \in O_{\alpha}(S)\) then \([A] \subseteq G\).

We denote by \(\alpha gc(S, \varphi, \tau, k)\), or when there can be no confusion by \(\alpha gc(S)\), the set of \(\alpha\)-g-closed sets in \(\varphi\).

Definition 1.6. [6] Let \((S, \varphi, \tau, k)\) be a complemented ditopological texture space. A subset \(A\) of a texture \(\varphi\) is called \(\alpha\)-g-open if \(\sigma(A)\) is \(\alpha\)-g-closed.

We denote by \(\alpha go(S, \varphi, \tau, k)\), or when there can be no confusion by \(\alpha go(S)\), the set of \(\alpha\)-g-open sets in \(\varphi\).

Definition 1.7. [6] Let \((S, \varphi, \tau, k, \sigma)\) be a complemented ditopological texture space. A subset \(A\) of a texture \(\varphi\) is called \(\alpha\)-g-closed if \(\alpha gc(S, \varphi, \tau, k)\), or when there can be no confusion by \(\alpha gc(S)\), the set of \(\alpha\)-g-closed sets in \(\varphi\) are called \(\alpha\)-g-interior \(\alpha go(S, \varphi, \tau, k)\), or when there can be no confusion by \(\alpha gc(S)\), the set of \(\alpha\)-g-open sets in \(\varphi\).

Corollary 2.2. Let \((f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)\) be a difunction. Then:

(1) Every continuous is \(\alpha\)-g-continuous.
(2) Every cocontinuous is \(\alpha\)-g-cocontinuous.
(3) Every \(\alpha\)-g-irresolute is \(\alpha\)-g-continuous.
(4) Every \(\alpha\)-g-co-irresolute is \(\alpha\)-g-cocontinuous.

Proof. Clear.

Theorem 2.3. Let \((f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)\) be a difunction. Then:

(1) The following are equivalent:
   (a) \((f, F)\) is \(\alpha\)-g-continuous.
   (b) \([F^{-\tau}A]^S_2 \subseteq [F^{-\tau}\alpha gc(S_1)]^S_2, \forall A \in \varphi_1\).
   (c) \(f^{-\tau}B^S_2 \subseteq [f^{-\tau}\alpha go(S_1)]^S_2, \forall B \in \varphi_2\).

(2) The following are equivalent:
   (a) \((f, F)\) is \(\alpha\)-g-cocontinuous.
   (b) \([F^{-\tau}A]^{\alpha gc(S_1)}^S_2 \subseteq [f^{-\tau}\alpha gc(S_1)]^S_2, \forall A \in \varphi_1\).
   (c) \([F^{-\tau}B]^{\alpha go(S_1)}^S_2 \subseteq [f^{-\tau}\alpha go(S_1)]^S_2, \forall B \in \varphi_2\).

Proof. We prove (1), leaving the dual proof of (2) to the interested reader.

\((a) \Rightarrow (b)\). Let \(A \in \varphi_1\). From [3] Theorem 2.24 (2a) and the definition of interior,

\[f^{-\tau}[F^{-\tau}(A)]^S_2 \subseteq f^{-\tau}(F^{-\tau}(A)) \subseteq A.\]

Since inverse image and co-image under a difunction is equal,

\[f^{-\tau}[F^{-\tau}(A)]^S_2 = F^{-\tau}[f^{-\tau}(A)]^S_2.\]
Thus,

\[ f^{-1}F^\rightarrow(A)^{S_2} \subseteq \alpha go(S_1), \]

by \( \alpha \)-\( g \)-continuity. Hence

\[ f^{-1}F^\rightarrow(A)^{S_2} \subseteq A_{\alpha-g}^{S_1} \]

and applying [3, Theorem 2.4 (2b)], gives

\[ |F^\rightarrow(A)^{S_2} \subset F^\rightarrow(f^{-1}F^\rightarrow(A)^{S_2}) \subseteq F^\rightarrow]A_{\alpha-g}^{S_1}, \]

which is the required inclusion.

(b) \( \Rightarrow \) (c). Take \( B \in \varphi_2 \). Applying inclusion (b) to \( A = f^{-1}(B) \) and using [3, Theorem 2.4 (2b)], gives

\[ |B|^{S_2} \subseteq F^\rightarrow(f^{-1}(B))^{S_2} \subseteq f^{-1}(B)|_{\alpha-g}^{S_1}. \]

Hence, we have

\[ f^{-1}|B|^{S_2} \subseteq f^{-1}|F^\rightarrow(f^{-1}(B))^{S_1}_{\alpha-g} \subseteq f^{-1}(B)|_{\alpha-g}^{S_1}. \]

by [3, Theorem 2.4 (2a)].

(c) \( \Rightarrow \) (a). Applying (c) for \( B \in O(S_2) \) gives

\[ f^{-1}(B) = f^{-1}|B|^{S_2} \subseteq f^{-1}(B)|_{\alpha-g}^{S_1}, \]

so

\[ F^\rightarrow(B) = f^{-1}(B) = |f^{-1}(B)|_{\alpha-g}^{S_1} \subseteq \alpha go(S_1). \]

Hence, \((f, F)\) is \( \alpha \)-\( g \)-continuous.

**Theorem 2.4.** Let \((f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \rightarrow (S_2, \varphi_2, \tau_2, k_2, \sigma_2)\) be a difunction. Then:

1. The following are equivalent:
   - (a) \((f, F)\) is \( \alpha \)-\( g \)- irresolute.
   - (b) \( |F^\rightarrow A|_{\alpha-g}^{S_2} \subseteq |F^\rightarrow A|_{\alpha-g}^{S_1}, \forall A \in \varphi_1. \)
   - (c) \( f^{-1}|B|_{\alpha-g}^{S_2} \subseteq f^{-1}|B|_{\alpha-g}^{S_1}, \forall B \in \varphi_2. \)

2. The following are equivalent:
   - (a) \((f, F)\) is \( \alpha \)-\( g \)-co- irresolute.
   - (b) \( |f^{-1}A|_{\alpha-g}^{S_1} \subseteq |f^{-1}A|_{\alpha-g}^{S_2}, \forall A \in \varphi_1. \)
   - (c) \( |F^\rightarrow B|_{\alpha-g}^{S_1} \subseteq |F^\rightarrow B|_{\alpha-g}^{S_2}, \forall B \in \varphi_2. \)

**Proof.** We prove (1), leaving the dual proof of (2) to the interested reader.

(a) \( \Rightarrow \) (b). Take \( A \in \varphi_1 \). Then:

\[ f^{-1}|F^\rightarrow A|_{\alpha-g}^{S_2} \subseteq f^{-1}(F^\rightarrow A) \subseteq A, \]

by [3, Theorem 2.4 (2a)]. Now

\[ f^{-1}|F^\rightarrow A|_{\alpha-g}^{S_2} = F^\rightarrow|F^\rightarrow A|_{\alpha-g}^{S_2} \in \alpha go(S_1) \]

by \( \alpha \)-\( g \)- irresolute, so

\[ f^{-1}|F^\rightarrow A|_{\alpha-g}^{S_2} \subseteq A|_{\alpha-g}^{S_1}, \]

and applying [3, Theorem 2.4 (2b)], gives

\[ |F^\rightarrow A|_{\alpha-g}^{S_2} \subseteq F^\rightarrow(f^{-1}|F^\rightarrow A|_{\alpha-g}^{S_2} \subseteq F^\rightarrow]A|_{\alpha-g}^{S_1}, \]

which is the required inclusion.
(b) $\Rightarrow$ (c). Take $B \in \varphi_2$. Applying inclusion (b) to $A = f^+B$ and using [3] Theorem 2.4 (2b), gives

$$|B|_{\alpha-g}^{S_2} \subseteq |F^+(f^+B)|_{\alpha-g}^{S_2} \subseteq F^+B|_{\alpha-g}^{S_1}.$$ 

Hence,

$$f^+|B|_{\alpha-g}^{S_2} \subseteq f^+F^+B|_{\alpha-g}^{S_1} \subseteq f^+B|_{\alpha-g}^{S_2}$$


(c) $\Rightarrow$ (a). Applying (c) for $B \in \alpha go(S_2)$ gives

$$f^+B = f^+|B|_{\alpha-g}^{S_2} \subseteq f^+B|_{\alpha-g}^{S_1},$$

so

$$F^+B = f^+B = |f^+B|_{\alpha-g}^{S_1} \in \alpha go(S_1).$$

Hence, $(f, F)$ is $\alpha$-g-irresolute.

**Theorem 2.5.** Let $(S_j, \varphi_j, \tau_j, k_j, \sigma_j)$, $j = 1, 2$, complemented ditopology and $(f, F): (S_1, \varphi_1) \rightarrow (S_2, \varphi_2)$ be complemented difunction. If $(f, F)$ is $\alpha$-g-continuous then $(f, F)$ is $\alpha$-g-cocontinuous.

**Proof.** Since $(f, F)$ is complemented, $(f^+, F^+) = (f, F)$. From [3] Lemma 2.20, $\sigma_1((f^+)\sigma_2(B)) = f^+(\sigma_2(B))$ and $\sigma_1((F^+)\sigma_2(B)) = F^+(\sigma_2(B))$ for all $B \in \varphi_2$. The proof is clear from these equalities.

**Corollary 2.6.** Let $(S_j, \varphi_j, \tau_j, k_j, \sigma_j)$, $j = 1, 2$, complemented ditopology and $(f, F): (S_1, \varphi_1) \rightarrow (S_2, \varphi_2)$ be complemented difunction. If $(f, F)$ is $\alpha$-g-irresolute then $(f, F)$ is $\alpha$-g-co-irresolute.

**Proof.** Clear.

**Definition 2.7.** A complemented ditopological texture space $(S, \varphi, \tau, k, \sigma)$ is called $\alpha$-g-compact if every cover of $S$ by $\alpha$-g-open has a finite subcover. Here we recall that $C = \{A_j : j \in J\}$, $A_j \in \varphi$ is a cover of $S$ if $\bigvee C = S$.

**Corollary 2.8.** Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space. Then:

1. Every $\alpha$-g-compact is compact.
2. Every $g$-compact is $\alpha$-g-compact.

**Proof.** Clear.

**Theorem 2.9.** If $(S, \varphi, \tau, k, \sigma)$ is $\alpha$-g-compact and $L = \{F_j : j \in J\}$ is a family of $\alpha$-g-closed sets with $\cap L = \phi$, then $\cap\{F_j : j \in J\} = \phi$ for $J' \subseteq J$ finite.

**Proof.** Suppose that $(S, \varphi, \tau, k, \sigma)$ is $\alpha$-g-compact and let $L = \{F_j : j \in J\}$ be a family of $\alpha$-g-closed sets with $\cap L = \phi$. Clearly, $C = \{\sigma(F_j) : j \in J\}$ is a family of $\alpha$-g-open sets. Moreover,

$$\bigvee C = \bigvee\{\sigma(F_j) : j \in J\} = \sigma(\cap\{F_j : j \in J\}) = \sigma(\phi) = \sigma(S),$$

and so we have $J' \subseteq J$ finite with $\bigvee\{\sigma(F_j) : j \in J'\} = \sigma(S)$. Hence $\cap\{F_j : j \in J'\} = \phi$.

**Theorem 2.10.** Let $(f, F): (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \rightarrow (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ be an $\alpha$-g-irresolute difunction. If $A \in \varphi_1$ is $\alpha$-g-compact then $f^+A \in \varphi_2$ is $\alpha$-g-compact.
Proof. Take \( f^{-1}A \subseteq \bigvee_{j \in J} G_j \), where \( G_j \in \alpha g(S_2) \), \( j \in J \). Now by [3, Theorem 2.24 (2a)], and [3, Corollary 2.12 (2)], we have
\[
A \subseteq F^+(f^{-1}A) \subseteq F^+(\bigvee_{j \in J} G_j) = \bigvee_{j \in J} F^+G_j.
\]
Also, \( F^+G_j \in \alpha g(S_1) \) because \((f, F)\) is \( \alpha g\)-irresolute. So by the \( \alpha g\)-compactness of \( A \) there exists \( J' \subseteq J \) finite such that \( A \subseteq \bigcup_{j \in J'} F^+G_j \). Hence
\[
f^{-1}A \subseteq f^{-1} \left( \bigcup_{j \in J'} F^+G_j \right) = \bigcup_{j \in J'} f^{-1}(F^+G_j) \subseteq \bigcup_{j \in J'} G_j
\]
by [3, Corollary 2.12 (2)], and [3, Theorem 2.24 (2b)]. This establishes that \( f^{-1}A \) is \( \alpha g\)-compact.

Corollary 2.11. Let \((f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \rightarrow (S_2, \varphi_2, \tau_2, k_2, \sigma_2)\) be a surjective \( \alpha g\)-irresolute difunction. Then, if \((S_1, \varphi_1, \tau_1, k_1, \sigma_1)\) is \( \alpha g\)-compact so is \((S_2, \varphi_2, \tau_2, k_2, \sigma_2)\).

Proof. This follows by taking \( A = S_1 \) in Theorem 2.10 and noting that \( f^{-1}S_1 = f^{-1}(F^+S_2) = S_2 \) by [3, Proposition 2.28 (1c)], and [3, Corollary 2.33 (1)].

Definition 2.12. A complemented ditopological texture space \((S, \varphi, \tau, k, \sigma)\) is called \( \alpha g\)-stable if very \( \alpha g\)-closed set \( F \in \varphi\{S\} \) is \( \alpha g\)-compact in \( S \).

Corollary 2.13. Let \((S, \varphi, \tau, k, \sigma)\) be a complemented ditopological texture space. Then:

1. Every \( \alpha g\)-stable is stable.
2. Every \( g\)-stable is \( \alpha g\)-stable.

Proof. Clear.

Theorem 2.14. Let \((S, \varphi, \tau, k, \sigma)\) be \( \alpha g\)-stable. If \( G \) is an \( \alpha g\)-open set with \( G \neq \phi \) and \( D = \{F_j : j \in J\} \) is a family of \( \alpha g\)-closed sets with \( \bigcap_{j \in J} F_j \subseteq G \) then \( \bigcap_{j \in J'} F_j \subseteq G \) for a finite subsets \( J' \) of \( J \).

Proof. Let \((S, \varphi, \tau, k, \sigma)\) be \( \alpha g\)-stable, let \( G \) be an \( \alpha g\)-open set with \( G \neq \phi \) and \( D = \{F_j : j \in J\} \) be a family of \( \alpha g\)-closed sets with \( \bigcap_{j \in J} F_j \subseteq G \). Set \( K = \sigma(G) \). Then \( K \) is \( \alpha g\)-closed and satisfies \( K \neq \emptyset \). Hence \( K \) is \( \alpha g\)-compact. Let \( C = \{\sigma(F) : F \in D\} \). Since \( \bigcap D \subseteq G \) we have \( K \subseteq \bigvee C \), that is \( C \) is an \( \alpha g\)-open cover of \( K \). Hence there exists \( F_1, F_2, \ldots, F_n \in D \) so that
\[
K \subseteq \sigma(F_1) \cup \sigma(F_2) \cup \ldots \cup \sigma(F_n) = \sigma(F_1 \cap F_2 \cap \ldots \cap F_n).
\]
This gives \( F_1 \cap F_2 \cap \ldots \cap F_n \subseteq \sigma(K) = G \), so \( \bigcap_{j \in J'} F_j \subseteq G \) for a finite subsets \( J' = \{1, 2, \ldots, n\} \) of \( J \).

Theorem 2.15. Let \((S_1, \varphi_1, \tau_1, k_1, \sigma_1), (S_2, \varphi_2, \tau_2, k_2, \sigma_2)\) be two complemented ditopological texture spaces with \((S_1, \varphi_1, \tau_1, k_1, \sigma_1)\) is \( \alpha g\)-stable, and \((f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \rightarrow (S_2, \varphi_2, \tau_2, k_2, \sigma_2)\) be an \( \alpha g\)-bi-irresolute difunction. Then \((S_2, \varphi_2, \tau_2, k_2, \sigma_2)\) is \( \alpha g\)-stable.

Proof. Take \( K \in \alpha g(S_2) \) with \( K \neq S_2 \). Since \((f, F)\) is \( \alpha g\)-co-irresolute, so \( f^{-1}K \in \alpha g(S_1) \). Let us prove that \( f^{-1}K \neq S_1 \). Assume the contrary. Since \( f^{-1}S_2 = S_1 \) by [3, Lemma 2.28 (1c)], we have \( f^{-1}S_2 \subseteq f^{-1}K \), whence \( S_2 \subseteq K \) by [3, Corollary 2.33 (1ii)], as \((f, F)\) is surjective. This is a contradiction, so \( f^{-1}K \neq S_1 \). Hence \( f^{-1}(K) \) is \( \alpha g\)-compact in \((S_1, \varphi_1, \tau_1, k_1, \sigma_1)\) by \( \alpha g\)-stability. As \((f, F)\) is \( \alpha g\)-irresolute, \( f^{-1}(f^{-1}K) \) is \( \alpha g\)-compact for the ditopology \((\tau_2, k_2)\) by Theorem 2.10 and by [3, Corollary 2.33 (1)], this set is equal to \( K \). This establishes that \((S_2, \varphi_2, \tau_2, k_2, \sigma_2)\) is \( \alpha g\)-stable.
References


