In this paper, we introduce new class of sets namely $\tau^*$-Generalized Locally Closed set, $\tau^*$-Generalized Semi Locally Closed set and functions namely $\tau^*$-glc-continuous function and $\tau^*$-gsl-continuous function in topological spaces and study some of their properties.

**Keywords:** $\tau^*$-generalized locally closed set; $\tau^*$-generalized semi locally closed set; $\tau^*$-glc-continuous function; $\tau^*$-gsl-continuous function.

**AMS Subject Classification:** 54A05.

### 1. Introduction


The purpose of this paper is to introduce and study the concept of new class of sets called $\tau^*$-Generalized Locally Closed set, $\tau^*$-Generalized Semi Locally Closed set and functions called $\tau^*$-glc-continuous function and $\tau^*$-gsl-continuous function in topological spaces and study some of their properties.

Throughout this paper $X$ and $Y$ are topological spaces on which no separation axioms are assumed unless otherwise explicitly stated. For a subset $A$ of a topological space $X$, $\text{Cl}(A)$, $\text{Cl}^*(A)$, $s\text{Cl}(A)$ and $Cl^*_\tau$ denote the closure, closure $*$, semi-closure and $\tau^*$-generalized closure of $A$ respectively.

### 2. Preliminaries

As we need the following definitions, let us recall them.
Definition 2.1 A subset $A$ of a topological space $(X, \tau)$ is called generalized closed [3] (briefly $g$-closed) in $X$ if $Cl(A) \subseteq G$ whenever $A \subseteq G$ and $G$ is open in $X$. A subset $A$ is called generalized open (briefly $g$-open) in $X$ if its complement $A^c$ is $g$-closed.

Definition 2.2 For the subset $A$ of a topological $X$, the generalized closure operator $Cl^*$ [5] is defined by the intersection of all $g$-closed sets containing $A$.

Definition 2.3 For the subset $A$ of a topological $X$, the topology $\tau^*$ [5] is defined by $\tau^* = \{G : Cl^*(GC) = GC\}$.

Definition 2.4 A subset $A$ of a topological space $X$ is called generalized semi closed set (briefly $gs$-closed) [6] if $sCl(A) \subseteq G$ whenever $A \subseteq G$ and $G$ is open in $X$. The complement of $gs$-closed set is called the generalized semi open set (briefly $gs$-open).

Definition 2.5 A subset $A$ of a topological space $X$ is called $\tau^*$-generalized closed set (briefly $\tau^*$-$g$-closed) [6] if $Cl^*(A) \subseteq G$ whenever $A \subseteq G$ and $G$ is $\tau^*$-open. The complement of $\tau^*$-generalized closed set is called the $\tau^*$-generalized open set (briefly $\tau^*$-$g$-open).

Definition 2.6 The $\tau^*$-generalized closure operator $Cl^*_\tau$ [5] for a subset $A$ of a topological space $(X, \tau^*)$ is defined by the intersection of all $\tau^*$-$g$-closed set containing $A$. That is

$$Cl^*_\tau(A) = \cap\{G : A \subseteq G \text{ and } G \text{ is } \tau^* - g - \text{closed}\}.$$ 

Definition 2.7 A topological space $(X, \tau^*)$ is called $\tau^*$-$Tg$ space [5] if every $\tau^*$-$g$-closed set in $X$ is $g$-closed in $X$.

Definition 2.8 A subset $S$ of a topological space $X$ is called locally closed [9] (briefly $LC$-closed) if $S = A \cap B$ where $A$ is open and $B$ is closed in $X$.

Definition 2.9 A subset $S$ of a topological space $X$ is called generalized locally closed set [4] (briefly GLC) if $S = A \cap B$ where $A$ is $g$-open and $B$ is $g$-closed in $X$.

Definition 2.10 A subset $S$ of a topological space $X$ is called GLC* [4] if $S = A \cap B$ where $A$ is $g$-open and $B$ is $g$-closed in $X$.

Definition 2.11 A subset $S$ of a topological space $X$ is called GLC** [4] if $S = A \cap B$ where $A$ is open and $B$ is $g$-closed in $X$.

Definition 2.12 A function $f : X \rightarrow Y$ from a topological space $X$ into a topological space $Y$ is called:

1. LC-continuous [9] if $f^{-1}(V) \in LC(X)$ for each open set $V$ in $Y$.
2. $\tau^*$-generalized continuous ($\tau^*$-$g$-continuous) [7] if the inverse image of every $g$-closed set in $Y$ is $\tau^*$-$g$-closed in $X$.
3. LC-irresolute [9] if $f^{-1}(V) \in LC(X)$ for each open set $V$ in $LC(X)$.
4. GLC-continuous [4] if $f^{-1}(V) \in GLC(X)$ for each open set $V$ in $Y$.
5. GLC-irresolute [4] if $f^{-1}(V) \in GLC(X)$ for each $V$ in $GLC(X)$.
6. GLC* -continuous [4] if $f^{-1}(V) \in GLC^*(X)$ for each $V$ in $Y$.
7. GLC** -continuous [4] if $f^{-1}(V) \in GLC^{**}(X)$ for each $V$ in $Y$.
8. GLC* -irresolute [4] if $f^{-1}(V) \in GLC^*(X)$ for each $V$ in $GLC^*(Y)$.
9. GLC** -irresolute [4] if $f^{-1}(V) \in GLC^{**}(X)$ for each $V$ in $GLC^{**}(Y)$.

Remark 2.13 In [6] it has been proved in Theorem 3.2 that every closed set is $\tau^*$-$g$-closed.

Remark 2.14 In [6] it has been proved in Theorem 3.4 that every $g$-closed set is $\tau^*$-$g$-closed.

**Notation:** $LC(X)$, $GLC(X)$, $GLC^*(X)$ and $GLC^{**}(X)$ denote the class of all locally closed sets, $glc$ sets, $glc^*$ sets and $glc^{**}$ sets respectively in a topological space $X$. 
3. \( \tau^*-\text{Generalized Locally Continuous Maps in Topological Spaces} \)

Using \( g \)-open set and \( g \)-closed set, K. Balachandran, P. Sundaram and H. Maki [4] introduced the concept of generalized locally closed set and generalized locally continuous maps. A. Pushpalatha, S. Eswaran and P. Rajarubi [6] introduced and studied a class of set namely \( \tau^* \)-generalized closed sets. In this section, we introduce a new class of set namely \( \tau^* \)-generalized locally closed set and a function namely \( \tau^*\text{-glc}-\text{continuous function in a topological space and study some of their properties.} \)

**Notation**: \( \tau^*-\text{GLC}(X) \) denotes the class of all \( \tau^*\text{-glc} \) sets in a topological space \( X \).

**Definition 3.1** A subset \( S \) of \( X \) is called \( \tau^* \)-generalized locally closed set (briefly \( \tau^*\text{-glc} \) set) if \( S = A \cap B \) where \( A \) is a \( \tau^*\text{-}g\)-open set and \( B \) is a \( \tau^*\text{-}g\)-closed set in \( X \).

**Theorem 3.2** If a subset \( S \) of \( X \) is locally closed, then it is \( \tau^*\text{-glc} \) but not conversely.

**Proof** Since \( S \) is locally closed, we can write \( S = A \cap B \), where \( A \) is \( g \)-open and \( B \) is \( g \)-closed in \( X \). By Remark 2.14, \( A \) is \( \tau^*\text{-}g\)-open and \( B \) is \( \tau^*\text{-}g\)-closed in \( X \). Hence \( S \) is \( \tau^*\text{-glc} \). \( \blacksquare \)

The converse of the theorem need not be true as seen from the following example.

**Example 3.3** Let \( X = \{a, b, c\} \), \( \tau = \{X, \phi, \{c\}\} \). Then \( S = \{a\} \) is \( \tau^*\text{-glc} \). But it is not locally closed.

**Theorem 3.4** A subset \( S \) of \( X \) is GLC if and only if it is \( \tau^*\text{-glc} \), provided \( X \) is a \( \tau^*\text{-Tg-space} \).

**Proof** Assume that \( S \) is GLC. Let \( S = A \cap B \) where \( A \) is \( g \)-open and \( B \) is \( g \)-closed in \( X \). By Remark 2.14, \( A \) is \( \tau^*\text{-}g\)-open and \( B \) is \( \tau^*\text{-}g\)-closed in \( X \). Thus, \( S \) is \( \tau^*\text{-glc} \). Conversely assume that \( S \) is \( \tau^*\text{-glc} \). Let \( S = A \cap B \) where \( A \) is \( \tau^*\text{-}g\)-open and \( B \) is \( \tau^*\text{-}g\)-closed in \( X \). Since \( X \) is a \( \tau^*\text{-Tg-space} \), \( A \) is \( g \)-open and \( B \) is \( g \)-closed in \( X \). Hence \( S \) is GLC. \( \blacksquare \)

**Theorem 3.5** If \( A \) is \( \tau^*\text{-glc} \) in \( X \) and \( B \) is \( \tau^*\text{-}g\)-open in \( X \) then \( A \cap B \) is \( \tau^*\text{-glc} \) in \( X \).

**Proof** Since \( A \) is \( \tau^*\text{-glc} \), we have \( A = P \cap Q \), where \( P \) is \( \tau^*\text{-}g\)-open and \( Q \) is \( \tau^*\text{-}g\)-closed in \( X \). Now

\[
A \cap B = (P \cap Q) \cap B = P \cap (Q \cap B) = P \cap (B \cap Q) = (P \cap B) \cap Q.
\]

Since \( P \) and \( B \) are \( \tau^*\text{-}g\)-open, \( P \cap B \) is also \( \tau^*\text{-}g\)-open and \( Q \) is \( \tau^*\text{-}g\)-closed. Hence \( A \cap B \) is \( \tau^*\text{-glc} \) in \( X \).

**Definition 3.6** A subset \( S \) of a topological space \( X \) is called \( \tau^*\text{-glc}^* \) if \( S = P \cap Q \) where \( P \) is \( \tau^*\text{-}g\)-open and \( Q \) is \( g \)-closed in \( X \).

**Definition 3.7** A subset \( S \) of a topological space \( X \) is called \( \tau^*\text{-glc}^{**} \) if \( S = P \cap Q \) where \( P \) is \( g \)-open and \( Q \) is \( \tau^*\text{-}g\)-closed in \( X \).

**Theorem 3.8** If \( A \) is \( \tau^*\text{-glc}^* \) in \( X \) and \( B \) is a \( \tau^*\text{-}g\)-open set in \( X \) then \( A \cap B \) is \( \tau^*\text{-glc}^* \) in \( X \).

**Proof** Since \( A \) is \( \tau^*\text{-glc}^* \), there exists a \( \tau^*\text{-}g\)-open set \( P \) and a \( g \)-closed set \( Q \) in \( X \) such that \( A = P \cap Q \). Now

\[
A \cap B = (P \cap Q) \cap B = (P \cap B) \cap Q.
\]

Since \( P \) and \( B \) are \( \tau^*\text{-}g\)-open, \( P \cap B \) is also \( \tau^*\text{-}g\)-open and \( Q \) is \( g \)-closed. Therefore \( A \cap B \) is \( \tau^*\text{-glc}^* \). \( \blacksquare \)

**Theorem 3.9** If \( A \) is \( \tau^*\text{-glc}^{**} \) in \( X \) and \( B \) is a \( g \)-closed set in \( X \) then \( A \cap B \) is \( \tau^*\text{-glc}^{**} \) in \( X \).

**Proof** Since \( A \) is \( \tau^*\text{-glc}^{**} \), we have \( A = P \cap Q \), where \( P \) is a \( g \)-open set and \( Q \) is a \( \tau^*\text{-}g\)-closed set in \( X \). Now

\[
A \cap B = (P \cap Q) \cap B = P \cap (Q \cap B).
\]
Given $B$ is $g$-closed. So by Remark 2.14, $B$ is $\tau^*-g$- closed. Therefore $Q \cap B$ is also $\tau^*-g$- closed. Hence $A \cap B$ is $\tau^*-glc^*$. ■

Theorem 3.10 A subset $A$ of a topological space $X$ is $\tau^*-glc^*$ if and only if there exists a $\tau^*-g$- open set $P$ such that $A = P \cap \text{cl}^* (A)$.

Proof Let $A$ be a $\tau^*-glc^*$. Then there exists a $\tau^*-g$- open set $P$ and a $g$- closed set $Q$ such that $A = P \cap Q$. Since $A \subset Q$ and $Q$ is $g$- closed, we have $A \subset \text{Cl}^*(A) \subset Q$. Also, $A \subset P$ and $A \subset \text{Cl}^*(A)$ together implies $A \subset P \cap \text{Cl}^*(A)$. On the other hand, take $x \in P \cap \text{Cl}^*(A)$. Then $x \in P$ and $x \in \text{Cl}^*(A) \subset Q$. So, $x \in P \cap Q = A$. Hence $P \cap \text{Cl}^*(A) \subset A$. Therefore $A = P \cap \text{Cl}^*(A)$.

Conversely assume that $A = P \cap \text{Cl}^*(A)$, where $P$ is $\tau^*-g$- open and $A$ is a subset of a topological space $X$. Here $\text{Cl}^*(A)$ is a $g$-closed set. Therefore $A$ is $\tau^*-glc^*$. ■

Theorem 3.11 If a subset $A$ of a topological space $X$ is $\tau^*-glc^*$, then there exists a $g$-open set $P$ such that $A = P \cap \text{Cl}^*(A)$.

Proof Let $A$ be a $\tau^*-glc^*$. By definition there exists a $g$-open set $P$ and a $\tau^*-g$-closed set $Q$ such that $A = P \cap Q$. Then, since $A \subset \text{Cl}^*(A) \subset Q$, we have $A \subset P \cap \text{Cl}^*(A)$. Also, if $x \in P \cap \text{Cl}^*(A)$, then $x \in Q$ and $x \in P$. Therefore $x \in P \cap Q = A$. Hence $P \cap \text{Cl}^*(A) \subset A$. Thus we have $A = P \cap \text{Cl}^*(A)$.

Theorem 3.12 If $A$ and $B$ are $\tau^*-glc^*$ in a topological space $X$, then $A \cap B$ is $\tau^*-glc^*$ in $X$.

Proof Since $A$ and $B$ are $\tau^*-glc^*$ sets, by Theorem 3.10, there exists $\tau^*-g$-open sets $P$ and $Q$ such that $A = P \cap \text{Cl}^*(A)$ and $B = Q \cap \text{Cl}^*(B)$. Therefore

$$A \cap B = (P \cap Q) \cap (\text{Cl}^*(A) \cap \text{Cl}^*(B)).$$

Since $P \cap Q$ is $\tau^*-g$-open and $\text{Cl}^*(A) \cap \text{Cl}^*(B)$ is $g$-closed, we have $A \cap B$ is $\tau^*-glc^*$. ■

Definition 3.13 A function $f : X \to Y$ from a topological space $X$ into to a topological space $Y$ is called:

1. $\tau^*-glc$-continuous if for each $g$-open set $V$ in $Y$, $f^{-1}(V)$ is $\tau^*-GLC(X)$.
2. $\tau^*-glc$-irresolute if for each $V \in \tau^*-GLC(Y)$, $f^{-1}(V) \in \tau^*-GLC(X)$.

Theorem 3.14 If a function $f : X \to Y$ is LC-continuous, then it is $\tau^*-glc$-continuous but not conversely.

Proof Assume that $f$ is LC-continuous. Let $V$ be an open set in $Y$. Then $f^{-1}(V)$ is locally closed in $X$. But by Theorem 3.2, every locally closed set is $\tau^*-glc$. Thus $f^{-1}(V)$ is $\tau^*-glc$ in $X$. Therefore $f$ is $\tau^*-glc$-continuous. ■

The converse of the above theorem need not be true as seen from the following example.

Example 3.15 Let $X = Y = \{a, b, c\}$, $\tau = \{X, \emptyset, \{c\}, \{a, b\}\}$ and $\sigma = \{Y, \emptyset, \{b\}, \{a, b\}\}$. Let $f : X \to Y$ be an identity map. Clearly $f$ is $\tau^*-glc$-continuous. But it is not LC-continuous since for the open set $V = \{b\}$ in $Y$, $f^{-1}(V)$ is not locally closed in $X$.

Theorem 3.16 If a function $f : X \to Y$ from a topological space $X$ into a topological space $Y$ is GLC-continuous then it is $\tau^*-glc$-continuous.

Proof Let $f : X \to Y$ be $glc$-continuous. Let $V$ be an open set in $Y$. Then $f^{-1}(V)$ is $glc$ set in $X$. Therefore by definition $f^{-1}(V) = A \cap B$, where $A$ is $g$-open and $B$ is $g$-closed in $X$. Since every open set is $g$-open, $V$ is $g$-open in $Y$. Also by Remark 2.14, $A$ and $B$ are $\tau^*-g$- open and $\tau^*-g$- closed respectively. Hence $f$ is $\tau^*-glc$-continuous. ■

Theorem 3.17 If a function $f : X \to Y$ from a topological space $X$ into a topological space $Y$ is $\tau^*-glc$-irresolute, then it is $\tau^*-glc$-continuous.
Proof Let $f : X \to Y$ be $\tau^*-\text{glc}$-irresolute. Let $V$ be a $g$-open set in $Y$. Since $g$-open implies $\tau^*-\text{glc}$, $V \in \tau^*\text{GLC}(Y)$. Also, since $f$ is $\tau^*-\text{glc}$-irresolute, $f^{-1}(V) \in \tau^*\text{GLC}(X)$. Therefore $f$ is $\tau^*-\text{glc}$-continuous.

Theorem 3.18 If a function $f : X \to Y$ from a topological space $X$ into a topological space $Y$ is $\tau^*-\text{glc}$-continuous and $A$ is a $\tau^*$-$g$-open subset of $X$, then the restriction $f/A : A \to Y$ is $\tau^*-\text{glc}$-continuous.

Proof Let $V$ be a $g$-open set in $Y$. Since $f$ is $\tau^*-\text{glc}$-continuous, $f^{-1}(V)$ is $\tau^*\text{glc}$ in $X$. We have $f^{-1}(V) = P \cap Q$, where $P$ is $\tau^*$-$g$-open and $Q$ is $\tau^*$-$g$-closed in $X$. Now

$$(f/A)^{-1}(V) = f^{-1}(V) \cap A = (P \cap A) \cap Q.$$ 

But $P \cap A$ is $\tau^*$-$g$-open in $X$ and therefore the restriction $f/A$ is $\tau^*$-$\text{glc}$-continuous. \[\blacksquare\]

Theorem 3.19

1. Let $f : X \to Y$ from a topological space $X$ into a topological space $Y$ is $\tau^*\text{glc}$-continuous and $B$ be a $g$-open subset of $Y$ containing $f(X)$. Then $f : X \to B$ is $\tau^*-\text{glc}$-continuous.
2. If $f : X \to Y$ and $g : Y \to Z$ are both $\tau^*-\text{glc}$-irresolute, then the composition $g \circ f : X \to Z$ is $\tau^*-\text{glc}$-irresolute.
3. If $f : X \to Y$ is $\tau^*-\text{glc}$-continuous and $g : Y \to Z$ is $\tau^*g$-continuous, then the composition $g \circ f : X \to Z$ is $\tau^*\text{glc}$-continuous.

Proof (1) Let $V$ be a $g$-open set in $B$. Since $B$ is a $g$-open subset of $Y$, the set $V$ is $g$-open in $Y$. And since $f$ is $\tau^*\text{glc}$-continuous, $f^{-1}(V)$ is $\tau^*\text{glc}$ in $X$. Therefore $f$ is $\tau^*-\text{glc}$-continuous be a $\tau^*\text{glc}$ in $Z$. Since $g$ is $\tau^*-\text{glc}$-irresolute, $g^{-1}(V)$ is $\tau^*\text{glc}$ in $Y$. Also, $f$ is $\tau^*\text{glc}$-irresolute. So, $f^{-1}(g^{-1}(V))$ is $\tau^*\text{glc}$ in $X$. But $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ and so $g \circ f$ is $\tau^*\text{glc}$-irresolute.

(2) Let $V$ be a $g$-open set in $Z$. Since $g$ is $\tau^*g$-continuous, $g^{-1}(V)$ is $\tau^*g$-open in $Y$. Also since $\tau^*\text{glc}$-continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ and so $g \circ f$ is $\tau^*\text{glc}$-continuous. \[\blacksquare\]

Remark 3.20 From the above discussion, we obtain the following implication.

$\text{LC-continuous} \Rightarrow \text{GLC-continuous} \Rightarrow \tau^*-\text{glc-continuous} \Rightarrow \tau^*-\text{glc-irresolute}$

A $\rightarrow$ B means A implies B, A $\not\rightarrow$ B means A does not imply B.

4. $\tau^*$-Generalized Semi Locally Closed sets and $\tau^*-\text{gsl}$-continuous Maps in Topological Spaces

In this section we introduce $\tau^*$-generalized semi locally closed set, $\tau^*-\text{gsl}$-continuous map and $\tau^*-\text{gsl}$-irresolute map and study some of their properties and relations with other maps.

Definition 4.1 A subset $A$ of $X$ is called $\tau^*$-generalized semi locally closed set (briefly $\tau^*-\text{gsl}$-closed) if $A = P \cap Q$, where $P$ is a $g$s-open set and $Q$ is a $g$s-closed in $X$.

Theorem 4.2 If a subset $S$ of $X$ is locally closed in $X$ then it is $\tau^*\text{gsl}$-closed in $X$ but not conversely.

Proof Assume that $S$ is locally closed in $X$. Then $S = A \cap B$ where $A$ is open and $B$ is closed in $X$. Since open set implies $gs$-open, we have $A$ is $gs$-open and $B$ is $gs$-closed in $X$. Therefore $S$ is $\tau^*\text{gsl}$-closed in $X$. \[\blacksquare\]
The converse of the above theorem need not be true as seen from the following example.

Example 4.3 Let \( X = \{a, b, c\} \), \( \tau = \{X, \phi, \{a\}, \{b, c\}\} \) and \( S = \{a, b\} \). Then \( S \) is not locally closed in \( X \) since it cannot be written as the intersection of an open set and a closed set in \( X \). But \( S \) is both \( gs \)-open and \( gs \)-closed in \( X \). Hence \( S \) is \( \tau^* \)-gsl-closed.

Definition 4.4 A function \( f : X \to Y \) is said to be \( \tau^* \)-gsl-continuous if the inverse image of every \( g \)-open set in \( Y \) is \( \tau^* \)-gsl-closed in \( X \).

Definition 4.5 A function \( f : X \to Y \) is said to be \( \tau^* \)-gsl-irresolute if the inverse image of every \( \tau^* \)-gsl-closed set in \( Y \) is \( \tau^* \)-gsl-closed in \( X \).

Theorem 4.6 If a function \( f : X \to Y \) is LC-continuous, then it is \( \tau^* \)-gsl-continuous but not conversely.

Proof Assume that \( f : X \to Y \) is LC-continuous. Then by definition, \( f^{-1}(V) \) is locally closed in \( X \), where \( V \) is open in \( Y \). Hence \( f^{-1}(V) = P \cap Q \), where \( P \) is open and \( Q \) is closed in \( X \). Since open \( \implies \) \( gs \)-open, we have \( P \) is \( gs \)-open and \( Q \) is \( gs \)-closed in \( X \). So \( f^{-1}(V) \) is \( \tau^* \)-gsl-closed in \( X \). Therefore \( f \) is \( \tau^* \)-gsl-continuous.

The converse of the above theorem need not be true as seen from the following example.

Example 4.7 Let \( X = Y = \{a, b, c\} \), \( \tau = \{X, \phi, \{c\}\} \), \( \sigma = \{Y, \phi, \{a\}, \{c\}, \{a, c\}, \{a, b\}, \{b, c\}\} \). Let \( f : X \to Y \) be an identity function. Then clearly \( f \) is \( \tau^* \)-gsl-continuous. But it is not LC-continuous since for the open set \( V = \{b, c\} \) in \( Y \), \( f^{-1}(V) = \{b, c\} \) is not locally closed in \( X \).

Theorem 4.8 If a function \( f : X \to Y \) is GLC-continuous, then it is \( \tau^* \)-gsl-continuous.

Proof Let \( f : X \to Y \) be glc-continuous. Let \( V \) be an open set in \( Y \). Since \( f \) is glc-continuous, \( f^{-1}(V) \) is glc in \( X \). Thus \( f^{-1}(V) \) can be written as \( P \cap Q \), where \( P \) is \( g \)-open and \( Q \) is \( g \)-closed. Since open \( \implies \) \( g \)-open \( \implies \) \( gs \)-open, \( P \) is \( gs \)-open and \( Q \) is \( gs \)-closed. Hence \( f^{-1}(V) \) is \( \tau^* \)-gsl-closed in \( X \). Therefore \( f \) is \( \tau^* \)-gsl-continuous.

The converse of the above theorem need not be true as seen from the following example.

Example 4.10 Let \( X = Y = \{a, b, c\} \), \( \tau = \{X, \phi, \{a, c\}\} \), \( \sigma = \{Y, \phi, \{a\}, \{a, b\}, \{a, c\}\} \). Let \( f : X \to Y \) be an identity function. Clearly \( f \) is \( \tau^* \)-gsl-irresolute. But it is not LC-continuous, because for the open set \( V = \{a\} \) in \( Y \), \( f^{-1}(V) = \{a\} \) is not locally closed in \( X \).

Remark 4.11 From the above discussion, we obtain the following implication.

\[
\begin{array}{c}
\text{LC-continuous} \\
\downarrow \\
\tau^* \text{-gsl-continuous} \\
\downarrow \\
\tau^* \text{-gsl-irresolute} \\
\end{array}
\]

\[
A \rightarrow B \text{ means } A \implies B, \ A \not\rightarrow B \text{ means } A \not\implies B.
\]
References