On intuitionistic supra $N$-closed set

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Abstract

The purpose of this paper is to introduce a new set called intuitionistic supra $N$-closed set on intuitionistic supra topological space. Also we investigate about the continuity and irresoluteness of the set on the intuitionistic supra topological space.

Keywords: ISNCS, intuitionistic supra $N$-continuity, intuitionistic supra $N$-irresolute map.


1. Introduction

Çoker [1, 3] introduced the concept of intuitionistic set and intuitionistic topological spaces. Zadeh [8] introduced the concept of fuzzy sets. Later on, Çoker [2] introduced the intuitionistic fuzzy topological spaces. Supra topology was introduced by Mashhour et al. [6]. Many authors have studied various sets on intuitionistic fuzzy topological spaces.

In this paper, we have introduced a new type of set called intuitionistic supra $N$-closed set on intuitionistic supra topological space and we have discussed about the continuity and irresoluteness of the set on the intuitionistic supra topological space.

2. Preliminaries

Definition 2.1. [1] Let $X$ be a non-empty set, an intuitionistic set (IS in short) $A$ is an object having the form $A = < X, A_1, A_2 >$, where $A_1$ and $A_2$ are subsets of $X$ satisfying $A_1 \cap A_2 = \phi$. The set $A_1$ is called the set of members of $A$, while $A_2$ is called the set of non-members of $A$.

Definition 2.2. [1] Let $X$ be a non-empty set, $A = < X, A_1, A_2 >$ and $B = < X, B_1, B_2 >$ be IS’s on $X$ and let $\{ A_i : i \in J \}$ be an arbitrary family of IS’s in $X$, where $A_i = < X, A_i^1, A_i^2 >$. Then:

1. $A \subseteq B$ iff $A_1 \subseteq B_1$ and $A_2 \supseteq B_2$.
2. $A = B$ iff $A \subseteq B$ and $B \subseteq A$. 

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Definition 2.3. [3] An intuitionistic topology on a non-empty set $X$ is a family $\tau$ of IS’s in $X$ satisfying the following axioms:

1. $X, \emptyset \in \tau$.
2. $A_1 \cap A_2 \in \tau$ for any $A_1, A_2 \in \tau$.
3. $\cup A_i \in \tau$ for any arbitrary family $\{A_i : i \in I\} \subseteq \tau$.

The pair $(X, \tau)$ is called an intuitionistic topological space (ITS in short) and IS in $\tau$ is known as an intuitionistic open set (IOS in short) in $X$, the complement of IOS is called an intuitionistic closed set (ICS in short) in $X$.

Definition 2.4. [3] Let $(X, \tau)$ be an ITS and let $A = \langle X, A_1, A_2 \rangle$ be an IS in $X$, then the interior and closure of $A$ are defined by:

$$\text{int}(A) = \bigcup \{ K : K \text{ is an IOS in } X \text{ and } A \subseteq K \}.$$ 

$$\text{cl}(A) = \bigcap \{ K : K \text{ is an ICS in } X \text{ and } A \subseteq K \}.$$ 

Definition 2.5. [1] Let $(X, \tau)$ and $(Y, \sigma)$ be two ITS’s and let $f : (X, \tau) \to (Y, \sigma)$ is said to be continuous iff the preimage of each IS in $\sigma$ is an IS in $\tau$.

Definition 2.6. [6] A subfamily $\mu$ of $X$ is said to be supra topology on $X$ if

1. $X, \emptyset \in \mu$.
2. If $A_i \in \mu, \forall i \in j$, then $\cup A_i \in \mu$.

The pair $(X, \mu)$ is called supra topological space. The element of $\mu$ are called supra open sets in $(X, \mu)$ and the complement of supra open sets is called supra closed sets and it is denoted by $\mu^c$.

Definition 2.7. [6] The supra closure of a set $A$ is denoted by $\text{cl}^\mu(A)$, and is defined as

$$\text{suprcl}(A) = \bigcap \{ B : B \text{ is supra closed and } A \subseteq B \}.$$ 

The supra interior of a set $A$ is denoted by $\text{int}^\mu(A)$, and is defined as

$$\text{suprint}(A) = \bigcup \{ B : B \text{ is supra open and } A \subseteq B \}.$$ 

Definition 2.8. [6] Let $(X, \tau)$ be a topological space and $\mu$ be a supra topology on $X$. We call $\mu$ a supra topology associated with $\tau$, if $\tau \subseteq \mu$.

Definition 2.9. [9] An intuitionistic supra topology on a non-empty set $X$ is a family $\tau$ of IS’s in $X$ satisfying the following axioms:

1. $X, \emptyset \in \tau$.
2. $\cup A_i \in \tau$ for any arbitrary family $\{A_i : i \in I\} \subseteq \tau$. 
Theorem 3.4. Every IS

Example 3.3. Let $X = (X, \tau)$ be an ISTS and let $A = (X, A_1, A_2)$ be an IS in $X$, then the supra closure and supra interior of $A$ are defined by:

$$cl^\sigma(A) = \bigcap \{K : K \text{ is an ISCS in } X \text{ and } A \subseteq K\},$$

$$int^\sigma(A) = \bigcup \{K : K \text{ is an ISOS in } X \text{ and } A \supseteq K\}.$$ 

Definition 2.10. [9] Let $(X, \tau)$ be an ISTS and let $A = (X, A_1, A_2)$ be an IS in $X$, then the supra closure and supra interior of $A$ are defined by:

$$(3) \text{ intutionistic supra}\ A_0\text{-closed set (ISNCS in short) if},$$

$$(4) \text{ intutionistic supra}\ A_0\text{-open set (ISNOS in short) if},$$

Theorem 3.2. Every ISCS is ISNCS.

Converse of the above theorem need not be true. It is shown by the following example.

Example 3.3. Let $X = (X, \tau) = \{X, A_1, A_2\}$, where $A_1 = (X, [a], [c])$ and $A_2 = (X, [a, b], \phi)$. Here $\langle X, [b, c], [a] \rangle$ is ISNCS but not ISCS.

Theorem 3.4. Every ISCS is ISNCS.
Theorem 4.2. Every intuitionistic supra continuous function $f : (X, \tau) \to (Y, \sigma)$ is called intuitionistic supra $N$-continuous if $f^{-1}(V)$ is ISNCS in $X$ for every ISCS $V$ in $Y$.

Theorem 4.4. Every intuitionistic supra continuous function is intuitionistic supra $N$-continuous function.
The converse of the above theorem need not be true. It is shown by the following example.

**Example 4.3.** Let \( X = Y = \{a, b, c\} \), \( \tau = \left\{X, \phi, A_1, A_2, A_3\right\} \), where \( A_1 = X, \{b\}, \{a, c\} \), \( A_2 = X, \{a\}, \{b\} \), and \( A_3 = X, \{a, b\}, \phi > \) and \( \sigma = \left\{Y, \phi, B_1, B_2\right\}, \) where \( B_1 = Y, \{a\}, \{c\} \), \( B_2 = Y, \{a, b\}, \phi > \). Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be the mapping defined by \( f(a) = a, f(b) = c \) and \( f(c) = b \). Here \( f \) is intutionistic supra \( N \)-continuous map but not intutionistic supra \( \Omega \)-continuous map, since \( V = Y, \phi, \{a, b\} > \) is ISCS in \( Y \) but \( f^{-1}(V) = X, \phi, \{a, c\} > \) is ISNCS but not ISCS in \( X \).

**Theorem 4.4.** Every intutionistic supra \( \alpha \) continuous map is intutionistic supra \( N \)-continuous map.

**Proof.** Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be an intutionistic supra \( \alpha \) continuous map. Let \( V \) be ISCS in \( Y \). Then \( f^{-1}(V) \) is ISaCS in \( X \). Since every ISaCS is ISNCS, then \( f^{-1}(V) \) is ISNCS in \( X \). Hence \( f \) is intutionistic supra \( N \)-continuous map.

The converse of the above theorem need not be true. It is shown by the following example.

**Example 4.5.** Let \( X = Y = \{a, b, c\} \), \( \tau = \left\{X, \phi, A_1, A_2, A_3\right\} \), where \( A_1 = X, \{b\}, \{a, c\} \), \( A_2 = X, \{a\}, \{b\} \), and \( A_3 = X, \{a, b\}, \phi > \), \( \sigma = \left\{Y, \phi, B_1, B_2\right\}, \) where \( B_1 = Y, \{a\}, \{c\} \), \( B_2 = Y, \{a, b\}, \phi > \). Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be the mapping defined by \( f(a) = a, f(b) = c \) and \( f(c) = b \). Here \( f \) is intutionistic supra \( N \)-continuous map but not intutionistic supra \( \alpha \)-continuous map, since \( V = Y, \phi, \{a, b\} > \) is ISCS in \( Y \) but \( f^{-1}(V) = X, \phi, \{a, c\} > \) is ISNCS but not ISaCS in \( X \).

**Theorem 4.6.** Every intutionistic supra semi continuous map is intutionistic supra \( N \)-continuous map.

**Proof.** Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be an intutionistic supra semi continuous map. Let \( V \) be ISCS in \( Y \). Then \( f^{-1}(V) \) is ISSCS in \( X \). Since every ISSCS is ISNCS, then \( f^{-1}(V) \) is ISNCS in \( X \). Hence \( f \) is intutionistic supra \( N \)-continuous map.

The converse of the above theorem need not be true. It is shown by the following example.

**Example 4.7.** Let \( X = Y = \{a, b, c\} \), \( \tau = \left\{X, \phi, A_1, A_2, A_3\right\}, \) where \( A_1 = X, \{b\}, \{a, c\} \), \( A_2 = X, \{a\}, \{b\} \), and \( A_3 = X, \{a, b\}, \phi > \), \( \sigma = \left\{Y, \phi, B_1, B_2\right\}, \) where \( B_1 = Y, \{a\}, \{c\} \), \( B_2 = Y, \{a, b\}, \phi > \). Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be the mapping defined by \( f(a) = a, f(b) = c \) and \( f(c) = b \). Here \( f \) is intutionistic supra \( N \)-continuous map but not intutionistic supra semi continuous map, since \( V = Y, \phi, \{a, b\} > \) is ISCS in \( Y \) but \( f^{-1}(V) = X, \phi, \{a, c\} > \) is ISNCS but not ISSCS in \( X \).

**Theorem 4.8.** Every intutionistic supra \( \Omega \) continuous map is intutionistic supra \( N \)-continuous map.

**Proof.** Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be an intutionistic supra \( \Omega \) continuous map. Let \( V \) be ISCS in \( Y \). Then \( f^{-1}(V) \) is ISQCS in \( X \). Since every ISQCS is ISNCS, then \( f^{-1}(V) \) is ISNCS in \( X \). Hence \( f \) is intutionistic supra \( N \)-continuous map.

The converse of the above theorem need not be true. It is shown by the following example.

**Example 4.9.** Let \( X = Y = \{a, b, c\} \), \( \tau = \left\{X, \phi, A_1, A_2\right\}, \) where \( A_1 = X, \{a\}, \{b\} \) and \( A_2 = X, \{a\}, \{b\} \), \( \phi > \) and \( \sigma = \left\{Y, \phi, B_1, B_2\right\}, \) where \( B_1 = Y, \{a\}, \{c\} \), \( B_2 = Y, \{a, b\}, \phi > \). Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be the mapping defined by \( f(a) = c, f(b) = b \) and \( f(c) = a \). Here \( f \) is intutionistic supra \( N \)-continuous map but not intutionistic supra \( \Omega \)-continuous map, since \( V = Y, \phi, \{a, b\} > \) is ISCS in \( Y \) but \( f^{-1}(V) = X, \phi, \{b, c\} > \) is ISNCS but not ISQCS in \( X \).
**Definition 4.10.** Let $(X, \tau)$ and $(Y, \sigma)$ be two intuitionistic supra topological space. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called intuitionistic supra $N$-irresolute map if $f^{-1}(V)$ is ISNCS in $X$ for every ISNCS $V$ in $Y$.

**Theorem 4.11.** Every intuitionistic supra $N$-irresolute map is intuitionistic supra $N$-continuous map.

**Proof.** Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic supra $N$-irresolute map. Let $V$ be ISCS in $Y$. Since every ISCS is ISNCS, then $V$ is ISNCS. Then $f^{-1}(V)$ is ISNCS in $X$. Hence $f$ is intuitionistic supra $N$-continuous map.

The converse of the above theorem need not be true. It is shown by the following example.

**Example 4.12.** Let $X = Y = \{a, b, c\}$. $\tau = \{X, \phi, A_1, A_2\}$, where $A_1 = \langle X, [a], [c]\rangle$ and $A_2 = \langle X, [a, b], \phi\rangle$. $\sigma = \{Y, \phi, B_1, B_2, B_3\}$, where $B_1 = \langle Y, [c]\rangle$, $B_2 = \langle Y, [a], [b]\rangle$ and $B_3 = \langle Y, [a], [b]\rangle$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the mapping defined by $f(a) = c$, $f(b) = a$ and $f(c) = b$. Here $f$ is intuitionistic supra $N$-continuous map but not intuitionistic supra $N$-irresolute map, since $V = \langle Y, \phi, [a, b]\rangle$ is ISNCS in $Y$ but $f^{-1}(V) = \langle X, \phi, [b, c]\rangle$ is not ISNCS in $X$.

**Theorem 4.13.** If $f : (X, \tau) \rightarrow (Y, \sigma)$ is intuitionistic supra $N$-continuous map and $g : (Y, \sigma) \rightarrow (Z, \eta)$ is intuitionistic supra continuous map then $g \circ f$ is intuitionistic supra $N$-continuous map.

**Proof.** Let $V$ be intuitionistic supra closed set in $Z$. Since $g$ is intuitionistic supra continuous map $g^{-1}(V)$ is intuitionistic supra closed set in $Y$. Since $f$ is intuitionistic supra $N$-continuous map, then $f^{-1}(g^{-1}(V))$ is intuitionistic supra $N$-closed set in $X$. Hence $g \circ f$ is intuitionistic supra $N$-continuous map.

**Theorem 4.14.** If $f : (X, \tau) \rightarrow (Y, \sigma)$ is intuitionistic supra $N$-irresolute map and $g : (Y, \sigma) \rightarrow (Z, \eta)$ is intuitionistic supra $N$-continuous map, then $g \circ f$ is intuitionistic supra $N$-continuous map.

**Proof.** Let $V$ be ISCS in $Z$. Since $g$ is intuitionistic supra $N$-continuous map $g^{-1}(V)$ is ISNCS in $Y$. Since $f$ is intuitionistic supra $N$-irresolute map, then $f^{-1}(g^{-1}(V))$ is ISNCS in $X$. Hence $g \circ f$ is intuitionistic supra $N$-continuous map.

**References**