Soft almost $\alpha$-continuous mappings

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Abstract

In the present paper the concept of soft almost $\alpha$-continuous mappings and soft almost $\alpha$-open mappings in soft topological spaces have been introduced and studied.

Keywords: Soft regular open set, Soft almost continuous mappings, Soft $\alpha$-continuous mappings, Soft almost $\alpha$-continuous mappings and Soft almost $\alpha$-open mappings.

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1. Introduction

In 1999 Molodtsov [8] introduced the concept of soft sets to deal with uncertainties while modeling the problems with incomplete information. In 2003, Maji [9] put forward several notions on Soft Set Theory. In 2011 Shabir and Naz [10] initiated the study of soft topological spaces. In the recent past many soft topological concept such as soft mappings, open sets and its weak form [1, 2, 12, 4, 10, 3, 13] have been introduced and studied. In the present paper we introduce the concept of soft almost $\alpha$-continuous and soft almost $\alpha$-open mappings and studied some of their properties and characterizations.

2. Preliminaries

Since we shall require the following known definitions, notations and some properties, we recall them in this section. Let $U$ is an initial universe set, $E$ be a set of parameters, $P(U)$ denote the power set of $U$ and $A \subseteq E$.

Definition 2.1. [8] A pair $(F, A)$ is called a soft set over $U$, where $F$ is a mapping given by $F: A \rightarrow P(U)$. In other words, a soft set over $U$ is a parameterized family of subsets of the universe $U$. For all $e \in A$, $F(e)$ may be considered as the set of $e$-approximate elements of the soft set $(F, A)$.

Definition 2.2. [9] For two soft sets $(F, A)$ and $(G, B)$ over a common universe $U$, we say that $(F, A)$ is a soft subset of $(G, B)$, denoted by $(F, A) \subseteq (G, B)$, if:

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(a) \( A \subseteq B \) and
(b) \( F(e) \subseteq G(e) \) for all \( e \in E \).

**Definition 2.3.** [9] Two soft sets \((F, A)\) and \((G, B)\) over a common universe \( U \) are said to be soft equal denoted by \((F, A) = (G, B)\) if \((F, A) \subseteq (G, B)\) and \((G, B) \subseteq (F, A)\).

**Definition 2.4.** [6] The complement of a soft set \((F, A)\), denoted by \( (F, A) \)\(^c \), is defined by \( (F, A) \)\(^c \) = \((F^c, A)\), where \( F^c : A \to P(U) \) is a mapping given by \( F^c(e) = U - F(e) \), for all \( e \in E \).

**Definition 2.5.** [9] Let a soft set \((F, A)\) over \( U \).
(a) Null soft set denoted by \( \phi \) if for all \( e \in A \), \( F(e) = \phi \).
(b) Absolute soft set denoted by \( \tilde{U} \), if for each \( e \in A \), \( F(e) = U \).

Clearly, \( \tilde{U} \)\(^c \) = \( \phi \) and \( \phi \)\(^c \) = \( \tilde{U} \).

**Definition 2.6.** [5] Union of two soft sets \((F, A)\) and \((G, B)\) over the common universe \( U \) is the soft set \((H, C)\), where \( C = A \cup B \), and for all \( e \in C \),
\[
H(e) = \begin{cases}  
F(e), & \text{if } e \in A - B \\
G(e), & \text{if } e \in B - A \\
F(e) \cup G(e), & \text{if } e \in A \cap B 
\end{cases}
\]

**Definition 2.7.** [5] Intersection of two soft sets \((F, A)\) and \((G, B)\) over a common universe \( U \), is the set \((H, C)\) where \( C = A \cap B \) and \( H(e) = F(e) \cap G(e) \) for each \( e \in E \).

Let \( X \) and \( Y \) be an initial universe sets and \( E \) and \( K \) be the non empty sets of parameters, \( S(X, E) \) denotes the family of all soft sets over \( X \) and \( S(Y, K) \) denotes the family of all soft sets over \( Y \).

**Definition 2.8.** [10] A subfamily \( \tau \) of \( S(X, E) \) is called a soft topology on \( X \) if:
(1) \( \tilde{\phi}, \tilde{X} \) belong to \( \tau \).
(2) The union of any number of soft sets in \( \tau \) belongs to \( \tau \).
(3) The intersection of any two soft sets in \( \tau \) belongs to \( \tau \).

The triplet \( (X, \tau, E) \) is called a soft topological space over \( X \). The members of \( \tau \) are called soft open sets in \( X \) and their complements called soft closed sets in \( X \).

**Definition 2.9.** If \( (X, \tau, E) \) is soft topological space and \( (F, E) \) be a soft set in \( X \) is said to be soft regular open set if \( (F, E) = \text{Int}(\text{Cl}(F, E)) \).

**Definition 2.10.** [3] The soft set \((F, E)\) \( \in S(X, E) \) is called a soft point if there exist \( x \in X \) and \( e \in E \) such that \( F(e) = \{x\} \) and \( F(e') = \phi \) for each \( e' \in E - \{e\} \), and the soft point \((F, E)\) is denoted by \((x, e)_E\).

**Definition 2.11.** [3] The soft point \((x, e)_E\) is said to be in the soft set \((G, E)\), denoted by \((x, e)_E \in (G, E)\) if \((x, e)_E \subseteq (G, E)\).

**Definition 2.12.** [12] Let \((X, \tau, E)\) be soft topological space and \((F, E)\) be a soft set in \( X \) is said to be soft regular open set if \((F, E) = \text{Int}(\text{Cl}(F, E))\).

**Definition 2.13.** [1] Let \((X, \tau, E)\) be soft topological space and \((F, E)\) be a soft set in \( X \) is said to be soft \( \alpha \)-open set if \((F, E) \subseteq \text{Int}(\text{Cl}(F, E))\).
Remark 2.14. [1] Every soft open (resp. closed) set is soft $\alpha$-open (resp. $\alpha$-closed) set but the converse may not be true.

Remark 2.15. [1] (1) Arbitrary union of soft $\alpha$-open sets is a soft $\alpha$-open set.
(2) Arbitrary intersection of soft $\alpha$-closed sets is a soft $\alpha$-closed set.

Definition 2.16. [1] If $(X, \tau, E)$ is soft topological space and a soft set $(F, E)$ over $X$.
(a) The soft $\alpha$-closure of $(F, E)$ is denoted by $\alpha \text{Cl}(F, E)$ is defined as the smallest soft $\alpha$-closed set over which contains $(F, E)$.
(b) The soft $\alpha$-interior of $(F, E)$ is denoted by $\alpha \text{Int}(F, E)$ is defined as the largest soft $\alpha$-open set over which is contained in $(F, E)$.

Definition 2.17. [4] Let $S(X,E)$ and $S(Y,K)$ be families of soft sets. Let $u: X \rightarrow Y$ and $p: E \rightarrow K$ be mappings. Then a mapping $f_{pu}: S(X, E) \rightarrow S(Y, K)$ is defined as :
(i) Let $(F, A)$ be a soft set in $S(X, E)$. The image of $(F, A)$ under $f_{pu}$, written as $f_{pu}(F, A) = (f_{pu}(F), p(A))$, is a soft set in $S(Y,K)$ such that
$$f_{pu}(F)(k) = \begin{cases} \bigcup_{e \in p^{-1}(k) \cap A} u(F(e)), & p^{-1}(k) \cap A \neq \emptyset \\ \phi, & p^{-1}(k) \cap A = \emptyset \end{cases}$$
For all $k \in K$.
(ii) Let $(G, B)$ be a soft set in $S(Y, K)$. The inverse image of $(G, B)$ under $f_{pu}$, written as
$$f_{pu}^{-1}(G)(e) = \begin{cases} u^{-1}G(p(e)), & p(e) \in B \\ \phi, & \text{otherwise} \end{cases}$$
For all $e \in E$.

Definition 2.18. [7] Let $f_{pu}: S(X, E) \rightarrow S(Y, K)$ be a mapping and $u: X \rightarrow Y$ and $p: E \rightarrow K$ be mappings. Then $f_{pu}$ is soft injective(resp. surjective,bijective) if $u: X \rightarrow Y$ and $p: E \rightarrow K$ are injective(resp. surjective,bijective).

Definition 2.19. [3] Let $(X,\tau,E)$ and $(Y,\nu,K)$ be soft topological spaces. A soft mapping $f_{pu}: (X,\tau,E)\rightarrow(Y,\nu,K)$ is called soft continuous if $f_{pu}^{-1}(G,K)$ is soft open in $X$, for every $(G,K)$ is soft open in $Y$.

Definition 2.20. [13] Let $(X,\tau,E)$ and $(Y,\nu,K)$ be soft topological spaces. A soft mapping $f_{pu}: (X,\tau,E)\rightarrow(Y,\nu,K)$ is called soft open if $f_{pu}(F, E)$ is soft open in $Y$, for all soft open set $(F, E)$ in $X$.

Definition 2.21. [11] Let $(X,\tau,E)$ and $(Y,\nu,K)$ be soft topological spaces. A soft mapping $f_{pu}: (X,\tau,E)\rightarrow(Y,\nu,K)$ is called soft almost continuous mapping if $f_{pu}^{-1}(G,K)$ is soft open in $X$, for all soft regular open set $(G,K)$ in $X$.

Definition 2.22. [11] Let $(X,\tau,E)$ and $(Y,\nu,K)$ be soft topological spaces. A soft mapping $f_{pu}: (X,\tau,E)\rightarrow(Y,\nu,K)$ is called soft almost open if $f_{pu}(F, E)$ is soft open in $Y$, for all soft regular open set $(F, E)$ in $X$.

Definition 2.23. [1] Let $(X,\tau,E)$ and $(Y,\nu,K)$ be soft topological spaces. A soft mapping $f_{pu}: (X,\tau,E)\rightarrow(Y,\nu,K)$ is called soft $\alpha$-continuous mapping if $f_{pu}^{-1}(G,K)$ is soft open in $X$, for all soft $\alpha$-open set $(G,K)$ in $Y$.

Definition 2.24. [1] Let $(X,\tau,E)$ and $(Y,\nu,K)$ be soft topological spaces. A soft mapping $f_{pu}: (X,\tau,E)\rightarrow(Y,\nu,K)$ is called soft $\alpha$-open if $f_{pu}(F, E)$ is soft open in $Y$, for all soft $\alpha$-open set $(F, E)$ in $X$. 
3. Soft Almost \( \alpha \)-Continuous Mappings

Definition 3.1. A soft mapping \( f_{pu} : (X, \tau, E) \rightarrow (Y, \delta, K) \) is said to be soft almost \( \alpha \)-continuous if the inverse image of every soft regular open set over \( Y \) is soft \( \alpha \)-open over \( X \).

Remark 3.2. Every soft almost continuous mapping is soft almost \( \alpha \)-continuous but converse may not be true.

Example 3.3. Let \( X = \{ x_1, x_2, x_3, x_4 \} \), \( E = \{ e_1, e_2 \} \) and \( Y = \{ y_1, y_2, y_3 \} \), \( K = \{ k_1, k_2 \} \). The soft sets \((E,E),(G_1,K),(G_2,K),(G_3,K)\) are defined as follows:

\[
\begin{align*}
F(e_1) &= \{ x_1 \}, & F(e_2) &= \emptyset, \\
G_1(k_1) &= \{ y_1 \}, & G_1(k_2) &= \emptyset, \\
G_2(k_1) &= \{ y_2 \}, & G_2(k_2) &= \emptyset, \\
G_3(k_1) &= \{ y_1, y_2 \}, & G_3(k_2) &= \emptyset
\end{align*}
\]

Let \( \tau = \{ \phi, (E,E), \tilde{X} \} \), and \( \nu = \{ \phi, (G_1,K),(G_2,K),(G_3,K), \tilde{Y} \} \) are topologies on \( X \) and \( Y \) respectively. Then soft mapping \( f_{pu} : (X, \tau, E) \rightarrow (Y, \nu, K) \) defined by \( u(x_1) = y_2 \), \( u(x_2) = y_3 \), \( u(x_3) = y_1 \) and \( p(e_1) = k_1 \), \( p(e_2) = k_2 \) is soft almost \( \alpha \)-continuous mapping not soft almost continuous.

Remark 3.4. Every soft \( \alpha \)-continuous mapping is soft almost \( \alpha \)-continuous but converse may not be true.

Example 3.5. Let \( X = \{ x_1, x_2, x_3 \} \), \( E = \{ e_1, e_2 \} \) and \( Y = \{ y_1, y_2, y_3 \} \), \( K = \{ k_1, k_2 \} \). The soft sets \((E,E),(G,K)\) are defined as follows:

\[
\begin{align*}
F(e_1) &= \{ x_2 \}, & F(e_2) &= \emptyset
\end{align*}
\]

Let \( \tau = \{ \phi, (E,E), \tilde{X} \} \), and \( \nu = \{ \phi, (G,K) \} \) are topologies on \( X \) and \( Y \) respectively. Then soft mapping \( f_{pu} : (X, \tau, E) \rightarrow (Y, \nu, K) \) defined by \( u(x_1) = y_2 \), \( u(x_2) = y_3 \), \( u(x_3) = y_1 \) and \( p(e_1) = k_1 \), \( p(e_2) = k_2 \) is soft almost \( \alpha \)-continuous but not soft \( \alpha \)-continuous.

Theorem 3.6. Let soft mapping \( f_{pu} : (X, \tau, E) \rightarrow (Y, \delta, K) \). Then the following conditions are equivalent:

(a) \( f_{pu} \) is soft almost \( \alpha \)-continuous.

(b) \( f_{pu}^{-1}(G,K) \) is soft \( \alpha \)-closed set in \( X \) for every soft regular closed set \((G,K)\) in \( Y \).

(c) \( f_{pu}^{-1}(A,K) \subset \alpha \text{Int}(f_{pu}^{-1}(\text{Int}(Cl(A,K)))) \) for every soft open set \((A,K)\) in \( Y \).

(d) \( \alpha Cl(f_{pu}^{-1}(\text{Int}(Cl(A,K)))) \subset f_{pu}^{-1}(G,K) \) for every soft closed set \((G,K)\) in \( Y \).

(e) For each soft point \((x_e)_E\) over \( X \) and each soft regular open set \((G,K)\) over \( Y \) containing \( f_{pu}((x_e)_E) \), there exists a soft \( \alpha \)-open set \((F,E)\) over \( X \) such that \((x_e)_E \in (F,E) \) and \((F,E) \subset f_{pu}^{-1}(G,K)\).

(f) For each soft point \((x_e)_E\) over \( X \) and each soft regular open set \((G,K)\) over \( Y \) containing \( f_{pu}((x_e)_E) \), there exists a soft \( \alpha \)-open set \((F,E)\) over \( X \) such that \((x_e)_E \in (F,E) \) and \( f_{pu}(F,E) \subset (G,K)\).

Proof: (a)\(\Rightarrow\)(b) Since \( f_{pu}^{-1}(G,K)^C = (f_{pu}^{-1}(G,K))^C \) for every soft set \((G,K)\) over \( Y \).

(a)\(\Rightarrow\)(c) Since \((A,K)\) is soft open set over \( Y \), \((A,K) \subset \text{Int}(Cl(A,K)) \) and hence, \( f_{pu}^{-1}(A,K) \subset f_{pu}^{-1}(\text{Int}(Cl(A,K))) \).

\( \text{Int}(Cl(A,K)) \) is a soft regular open set over \( Y \). Hence \( f_{pu}^{-1}(\text{Int}(Cl(A,K))) \) is soft \( \alpha \)-open set over \( X \). Thus, \( f_{pu}^{-1}(A,K) \subset f_{pu}^{-1}(\text{Int}(Cl(A,K))) = \alpha \text{Int}(f_{pu}^{-1}(\text{Int}(Cl(A,K)))) \).

(c)\(\Rightarrow\)(a) Let \((A,K)\) be a soft regular open set over \( Y \), then we have \( f_{pu}^{-1}(A,K) \subset \alpha \text{Int}(f_{pu}^{-1}(\text{Int}(Cl(A,K)))) \).

Thus, \( f_{pu}^{-1}(A,K) = \alpha \text{Int}(f_{pu}^{-1}(A,K)) \) shows that \( f_{pu}^{-1}(A,K) \) is a soft \( \alpha \)-open set over \( X \).

(b)\(\Rightarrow\)(d) Since \((G,K)\) is soft closed set over \( Y \), \( \text{Cl}(\text{Int}(G,K)) \subset (G,K) \) and \( f_{pu}^{-1}(\text{Cl}(\text{Int}(G,K))) \subset f_{pu}^{-1}(G,K) \).

\( \text{Cl}(\text{Int}(G,K)) \) is soft regular closed set over \( Y \). Hence, \( f_{pu}^{-1}(\text{Cl}(\text{Int}(G,K))) \) is soft \( \alpha \)-closed set over \( X \). Thus, \( \alpha Cl(f_{pu}^{-1}(\text{Cl}(\text{Int}(G,K)))) = f_{pu}^{-1}(\text{Cl}(\text{Int}(G,K))) \subset f_{pu}^{-1}(G,K) \).

(d)\(\Rightarrow\)(b) Let \((G,K)\) be a soft regular closed set over \( Y \), then we have \( \alpha \text{Cl}(f_{pu}^{-1}(G,K)) = \alpha Cl(f_{pu}^{-1}(\text{Cl}(\text{Int}(G,K)))) \subset f_{pu}^{-1}(G,K) \).

Thus, \( \alpha Cl(f_{pu}^{-1}(G,K)) \subset f_{pu}^{-1}(G,K) \), shows that \( f_{pu}^{-1}(G,K) \) is soft \( \alpha \)-closed set over \( X \).
(a)⇒(e) Let \((x_e)\) be a soft point over \(X\) and \((G,K)\) be a soft regular open set over \(Y\) such that \(f_{pu}(x_e) \in (G,K)\). Put \((F,E) = f_{pu}^{-1}(G,K)\). Then by (a), \((F,E)\) is soft 0-open set, \((x_e) \in (F,E)\) and \((F,E) \subset f_{pu}^{-1}(G,K)\).

(e)⇒(f) Let \((x_e)\) be a soft point over \(X\) and \((G,K)\) be a soft regular open set over \(Y\) such that \(f_{pu}(x_e) \in (G,K)\). By (e) there exists a soft 0-open set \((F,E)\) such that \((x_e) \in (F,E)\), \((F,E) \subset f_{pu}^{-1}(G,K)\). Then \(f_{pu}(x_e) \in f_{pu}(F,E) \subset G(K)\). By (f) there exists a soft 0-open set \((F,E)\) such that \((x_e) \in (F,E)\) and \(f_{pu}(F,E) \subset (G,K)\). This shows that \((x_e) \in (F,E) \subset f_{pu}^{-1}(G,K)\). It follows that \(f_{pu}^{-1}(G,K)\) is soft 0-open set and hence \(f_{pu}^{-1}\) is soft almost 0-continuous.

**Definition 3.7.** A soft topological space \((X,\tau,E)\) is said to be soft semiregular if for each soft open set \((F,E)\) and each soft point \((x_e) \in (F,E)\), there exists a soft open set \((G,K)\) such that \((x_e) \in (G,K)\) and \((G,K) \subset \text{Int}(\text{Cl}(G,K)) \subset (F,E)\).

**Theorem 3.8.** Let \(f_{pu} : (X,\tau,E) \rightarrow (Y,\delta,K)\) be a soft mapping from a soft topological space \((X,\tau,E)\) to a soft semiregular space \((Y,\delta,K)\). Then \(f_{pu}\) is soft almost 0-continuous if and only if \(f_{pu}\) is soft 0-continuous.

**Proof:** Suppose \((x_e)\) is a soft point in \(X\) and \((E,K)\) is a soft open set in \(Y\) such that \(f_{pu}(x_e) \in (E,K)\). Since \((Y,\delta,K)\) is soft semiregular there exists a soft open set \((G,K)\) in \(Y\) such that \(f_{pu}(x_e) \in (G,K)\) and \((G,K) \subset \text{Int}(\text{Cl}(G,K)) \subset (E,K)\). If \(f_{pu}\) is soft almost 0-continuous, there exists a soft 0-open set \((A,E)\) in \(X\) such that \((x_e) \in (A,E)\) and \(f_{pu}(A,E) \subset \text{Int}(\text{Cl}(G,K))\). Thus, \((A,E)\) is soft 0-open set such that \((x_e) \in (A,E)\) and \(f_{pu}(A,E) \subset (F,E)\). Hence, \(f_{pu}\) is soft 0-continuous.

**Theorem 3.9.** If soft mapping \(f_{pu}^{-1} : (X,\tau,E) \rightarrow (Y,\delta,K)\) is soft open soft continuous and soft mapping \(g_{pu}^{-1} : (Y,\delta,K) \rightarrow (Z,\eta,T)\) is soft almost 0-continuous, then \(g_{pu}^{-1} \circ f_{pu}^{-1} : (X,\tau,E) \rightarrow (Z,\eta,T)\) is soft almost 0-continuous.

**Proof:** Suppose \((U,T)\) is a soft regular open set in \(Z\). Then \(g_{pu}^{-1}(U,T)\) is a soft 0-open set in \(Y\) because \(g_{pu}^{-1}\) is soft almost 0-continuous. And so \(f_{pu}^{-1}\) being soft open and continuous. \((f_{pu}^{-1} \circ g_{pu}^{-1}(U,T))\) is 0-open in \(X\). Consequently, \(g_{pu}^{-1} \circ f_{pu}^{-1} : (X,\tau,E) \rightarrow (Z,\eta,T)\) is soft almost 0-continuous.

**Theorem 3.10.** Let \(f_{pu} : (X,\tau,E) \rightarrow (Y,\delta,K)\) be a soft almost 0-continuous mapping and \((A,E)\) is soft open set in \(X\). Then \(f_{pu}(A,E)\) is soft almost 0-continuous.

**Proof:** Let \((G,K)\) be a soft regular open set in \(Y\) then \(f_{pu}^{-1}(G,K)\) is soft 0-open in \(X\). Since \((A,E)\) is soft open in \(X\), \((A,E) \cap f_{pu}^{-1}(G,K) = [f_{pu}(A,E)]^{-1}(G,K)\) is soft 0-open in \(A,E\). Therefore, \(f_{pu}(A,E)\) is soft almost 0-continuous.

4. **Soft Almost 0-Open Mappings**

**Definition 4.1.** A soft mapping \(f_{pu} : (X,\tau,E) \rightarrow (Y,\delta,K)\) is said to be soft almost 0-open if for each soft regular open set \((F,E)\) in \(X\), \(f_{pu}(F,E)\) is soft 0-open in \(Y\).

**Remark 4.2.** Every soft almost open is soft almost 0-open but converse may not be true.

**Example 4.3.** Let \(X = \{x_1, x_2\}, E = \{e_1, e_2\}\) and \(Y = \{y_1, y_2\}, K = \{k_1, k_2\}\). The soft sets \((F_1,E), (F_2,E)\) \((G,K)\) are defined as follows:

\[
F_1(e_1) = X, \\
F_1(e_2) = \{x_1\}, \\
F_2(e_1) = \phi, \\
F_2(e_2) = \{x_2\}, \\
G(k_1) = \phi, \\
G(k_2) = \{y_1\}.
\]

Let \(\tau = \{\phi, (F_1,E), (F_2,E), \tilde{X}\}\) and \(\nu = \{\phi, (G,K), \tilde{Y}\}\) are topologies on \(X\) and \(Y\) respectively. Then soft mapping \(f_{pu} : (X,\tau,E) \rightarrow (Y,\nu,K)\) defined by \(u(x_1) = y_1, u(x_2) = y_2\) and \(p(e_1) = k_1, p(e_2) = k_2\) is soft almost 0-open mapping but not soft almost open.
Remark 4.4. Every soft $\alpha$-open mappings is soft almost $\alpha$-open but converse may not be true.

Example 4.5. Let $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $Y = \{y_1, y_2\}$, $K = \{k_1, k_2\}$. The soft sets $(F,E)$ is defined as follows:

\[ F(e_1) = \{x_1\}, \quad F(e_2) = \emptyset. \]

Let $\tau = \{\phi, (F,E), \tilde{X}\}$, and $\nu = \{\phi, \tilde{Y}\}$ are topologies on $X$ and $Y$ respectively. Then soft mapping $f_{\nu u}: (X, \tau, E) \rightarrow (Y, \nu, K)$ defined by $u(x_1) = y_1$, $u(x_2) = y_2$ and $p(e_1) = k_1$, $p(e_2) = k_2$. $k_2$ is soft almost $\alpha$-open mapping but not soft $\alpha$-open.

Theorem 4.6. Let $f_{\nu u_1}: (X, \tau, E) \rightarrow (Y, \nu, K)$ and $g_{\nu u_2}: (Y, \nu, K) \rightarrow (Z, \eta, T)$ be two soft mappings. If $f_{\nu u_1}$ is soft almost open and $g_{\nu u_2}$ is soft $\alpha$-open. Then the soft mapping $g_{\nu u_2} \circ f_{\nu u_1}: (X, \tau, E) \rightarrow (Z, \eta, T)$ is soft almost $\alpha$-open.

Proof: Let $(F,E)$ be soft regular open in $X$. Then $f_{\nu u_1}(F,E)$ is soft open in $Y$ because $f_{\nu u_1}$ is soft almost open. Therefore, $g_{\nu u_2}(f_{\nu u_1})(F,E)$ is soft $\alpha$-open in $Z$. Because $g_{\nu u_2}$ is soft $\alpha$-open. Since $(g_{\nu u_2} \circ f_{\nu u_1})(F,E) = (g_{\nu u_2}(f_{\nu u_1}(F,E)))$, it follows that the soft mapping $(g_{\nu u_2} \circ f_{\nu u_1})$ is soft almost $\alpha$-open.

Definition 4.7. A soft mapping $f_{\nu u}: (X, \tau, E) \rightarrow (Y, \nu, K)$ is said to be soft $\alpha$-irresolvent if the inverse image of soft open set of $Y$ is soft $\alpha$-open set in $X$.

Theorem 4.8. Let $f_{\nu u_1}: (X, \tau, E) \rightarrow (Y, \nu, K)$ and $g_{\nu u_2}: (Y, \nu, K) \rightarrow (Z, \eta, T)$ be two soft mappings, such that $g_{\nu u_2} \circ f_{\nu u_1}: (X, \tau, E) \rightarrow (Z, \eta, T)$ is soft almost $\alpha$-open and $g_{\nu u_2}$ is soft $\alpha$-irresolvent and injective then $f_{\nu u_1}$ is soft almost $\alpha$-open.

Proof: Suppose $(F,E)$ is soft regular open set in $X$. Then $g_{\nu u_2} \circ f_{\nu u_1}(F,E)$ is soft $\alpha$-open in $Z$ because $g_{\nu u_2} \circ f_{\nu u_1}$ is soft almost $\alpha$-open. Since $g_{\nu u_2}$ is injective, we have $(g_{\nu u_2}^{-1}(g_{\nu u_2} \circ f_{\nu u_1})(F,E)) = f_{\nu u_1}(F,E)$. Therefore $f_{\nu u_1}(F,E)$ is soft $\alpha$-open in $Y$, because $g_{\nu u_2}$ is soft $\alpha$-irresolvent. This implies $f_{\nu u_1}$ is soft almost $\alpha$-open.

Theorem 4.9. Let soft mapping $f_{\nu u}: (X, \tau, E) \rightarrow (Y, \nu, K)$ be soft almost $\alpha$-open mapping. If $(G,K)$ is soft set of $Y$ and $(F,E)$ is soft regular closed set of $X$ containing $f_{\nu u}^{-1}(G,K)$ then there is a soft $\alpha$-closed set $(A,K)$ of $Y$ containing $(G,K)$ such that $f_{\nu u}^{-1}(A,K) \subset (F,E)$.

Proof: Let $(A, K) = (f_{\nu u}(F,E))^C$. Since $f_{\nu u}^{-1}(G,K) \subset (F,E)$ we have $f_{\nu u}(F,E)^C \subset (G,K)$. Since $f_{\nu u}$ is soft almost $\alpha$-open then $(A,K)$ is soft $\alpha$-closed set of $Y$ and $f_{\nu u}^{-1}(A,K) = (f_{\nu u}^{-1}(f_{\nu u}(F,E)^C))^C \subset ((F,E)^C)^C = (F,E)$. Thus, $f_{\nu u}^{-1}(A,K) \subset (F,E)$.

References


