Journal of Advanced Studies in Topology
eISSN: 2090-388X
pISSN: 2090-8288
Vol. 4, No. 1, 2013, 73–79

RESEARCH ARTICLE

γ*gb-Closed Sets in Topological Spaces*

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(*Received: 3 September 2012, Accepted: 30 October 2012*)

In this paper, we introduce a new class of sets called γ-generalized b-closed sets in topological spaces (briefty γgb-closed set). Also we study some of its basic properties and investigate the relations between the associated topology.

Keywords: γ-open sets; γgb-closed sets.

AMS Subject Classification: 54A40.

1. Introduction

Vidhya and Parimelazhagan [1] introduced a new class of generalized closed sets in a topological space its called g^*b-closed sets. Levine [2] introduced the notion of generalized closed (briefty g-closed) sets in topological spaces and showed that compactness, countably compactness, para compactness and normality etc are all g-closed hereditary. Dontchev [3], Maki, Devi and Balachandran [4], Mashhour, Abd-El-Monsef and El-Deeb [5], Andrijevic [6] and Nagaveni [7] introduced and investigated the concept of generalized semi-preclosed sets, generalized α-closed sets, preclosed sets, semi-preclosed sets and weakly generalized closed sets respectively. Andrijevic [8], introduced a class of generalized open sets in a topological space called b-open sets. Omari and Noorani [9] introduced and studied the concept of generalized b-closed sets (briefty gb-closed) in topological spaces.

In this paper, we introduce a new class of sets called γgb-closed set and study some of its basic properties and investigate the relations between the associated topology.

2. Preliminaries

Throughout this paper, (X, τ) and (Y, σ) stand for topological spaces with no separation axioms assumed unless otherwise stated. For a subset A of X, the closure of A and the interior of A will be denoted by $cl(A)$ and $int(A)$, respectively.

Let (X, τ) be a topological space and A a subset of X. An operation γ [10] on a topology τ is a mapping from τ in to power set $P(X)$ of X such that $V \subset \gamma(V)$ for each $V \in \tau$, where $\gamma(V)$ denotes the value of γ at V. A subset A of X with an operation γ on τ is called γ-open [10] if for each $x \in A$, there exists an open set U such that $x \in U$ and $\gamma(U) \subset A$. Then, τ_γ denotes the set of all γ-open set in X. Clearly $\tau_\gamma \subset \tau$. Complements of γ-open sets are called γ-closed. The γ-closure [10] of a subset A of X with an operation γ on τ is denoted by τ_γ-$cl(A)$ and is defined to be the intersection of all γ-closed sets containing A. A topological X with an operation γ on τ is said to be γ-regular [10] if for

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each $x \in X$ and for each open neighborhood V of x, there exists an open neighborhood U of x such that $\gamma(U)$ contained in V. It is also to be noted that $\tau = \tau_\gamma$ if and only if X is a γ-regular space [10].

Definition 2.1 [5] A subset A of a topological space (X, τ) is called a pre-open set if $A \subseteq int(cl(A))$ and preclosed set if $cl(int(A)) \subseteq A$.

Definition 2.2 [11] A subset A of a topological space (X, τ) is called a semi-open set if $A \subseteq cl(int(A))$ and semi-closed set if $int(cl(A)) \subseteq A$.

Definition 2.3 [12] A subset A of a topological space (X, τ) is called an α-open set if $A \subseteq int(cl(int(A)))$ and an α-closed set if $cl(int(cl(A))) \subseteq A$.

Definition 2.4 [6] A subset A of a topological space (X, τ) is called a semi-preopen set (β-open set) if $A \subseteq cl(int(cl(A)))$ and semi-preclosed set if $int(cl(int(A))) \subseteq A$.

Definition 2.5 [8] A subset A of a topological space (X, τ) is called a b-open set if $A \subseteq cl(int(A)) \cup int(cl(A))$ and b-closed set if $cl(int(A)) \cup int(cl(A)) \subseteq A$.

Definition 2.6 [2] A subset A of a topological space (X, τ) is called a generalized closed set (briefty g-closed) if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in X.

Definition 2.7 [4] A subset A of a topological space (X, τ) is called a generalized α-closed (briefty gα-closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in X.

Definition 2.8 [13] A subset A of a topological space (X, τ) is called an α-generalized closed (briefty αg-closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

Definition 2.9 [14] A subset A of a topological space (X, τ) is called a generalized preclosed (briefty gp-closed) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

Definition 2.10 [15] A subset A of a topological space (X, τ) is called a generalized semiclosed (briefty gs-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

Definition 2.11 [16] A subset A of a topological space (X, τ) is called a generalized semiclosed (briefty sg-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X.

Definition 2.12 [9] A subset A of a topological space (X, τ) is called a generalized b-closed set (briefty gb-closed) if $bcl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in X.

Definition 2.13 [7] A subset A of a topological space (X, τ) is called a weakly generalized closed set (briefty wg-closed) if $cl(int(A)) \subseteq U$, whenever $A \subseteq U$ and U is open in X.

Definition 2.14 [3] A subset A of a topological space (X, τ) is called a generalized semi-preclosed set (briefty gsp-closed) if $spcl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in X.

Definition 2.15 [17] A subset A of a topological space (X, τ) is called a generalized *closed set (briefty g^*-closed) if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is g-open in X.

Definition 2.16 [1] A subset A of a topological space (X, τ) is called a g^*b-closed set if $bcl(A) \subseteq U$, whenever $A \subseteq U$ and U is g-open in X.

3. γgb-Closed Sets

In this section we introduce the concept of γgb-closed sets in topological space and we investigate the group of structure of the set of all γgb-closed sets.

Definition 3.1 A subset A of a topological space (X, τ) is called a γgb-closed set if $bcl(A) \subseteq U$, whenever $A \subseteq U$ and U is γ-open in X.

Theorem 3.2 Every γ-closed set is γgb-closed set.

Proof Let A be a γ-closed set in X such that $A \subseteq U$, where U is γ-open. Since A is γ-closed, τ_γ-$cl(A) = A$. Since $bcl(A) \subseteq cl(A) \subseteq \tau_\gamma$-$cl(A) = A$. Therefore $bcl(A) \subseteq U$. Hence A is a γgb-closed set in X. ■

The converse of the above theorem need not be true as seen from the following example.

Example 3.3 Let $X = \{a, b, c\}$ with the topology $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Define an operation γ on τ by

$$\gamma(A) = \begin{cases} A & \text{if } A = \{b\} \text{ or } \{a, c\}, \\ X & \text{otherwise.} \end{cases}$$

Let $A = \{a\}$, since the only γ-open supersets of A are $\{a, c\}$ and X, then A is a γgb-closed set but not a γ-closed set of (X, τ).

Theorem 3.4 Every closed set is γgb-closed set.

Proof Let A be a closed set in X such that $A \subseteq U$, where U is γ-open. Since A is closed, $cl(A) = A$. Since $bcl(A) \subseteq cl(A) = A$. Therefore $bcl(A) \subseteq U$. Hence A is a γgb-closed set in X. ■

The converse of the above theorem need not be true as seen from the following example.

Example 3.5 Let $X = \{a, b, c\}$ with the topology $\tau = \{X, \phi, \{b\}, \{b, c\}\}$ and $\gamma(A) = A$ for all $A \in \tau$. Let $A = \{a, b\}$. Here A is a γgb-closed set but not a closed set of (X, τ).

Theorem 3.6 Every b-closed set is γgb-closed set.

Proof Let A be a b-closed set in X such that $A \subseteq U$, where U is γ-open. Since A is b-closed, $bcl(A) = A$. Therefore $bcl(A) \subseteq U$. Hence A is a γgb-closed set in X. ■

The converse of the above theorem need not be true as seen from the following example.

Example 3.7 Let $X = \{a, b, c\}$ with the topology $\tau = \{X, \phi, \{b\}, \{a, b\}\}$ and $\gamma(A) = A$ for all $A \in \tau$. Let $A = \{b, c\}$. Here A is a γgb-closed set but not a b-closed set of (X, τ).

Theorem 3.8 Every gb-closed set is γgb-closed set.

Proof Let A be a gb-closed set in X such that $A \subseteq U$, where U is γ-open. Since every γ-open set is open and A is gb-closed, $bcl(A) \subseteq U$. Hence A is γgb-closed. ■

The converse of the above theorem need not be true as seen from the following example.

Example 3.9 From Example 3.3, let $A = \{a\}$. Here A is a γgb-closed set but not a gb-closed set of (X, τ).

Theorem 3.10 Every α-closed set is γgb-closed set.

Proof Let A be a α-closed set in X such that $A \subseteq U$, where U is γ-open. Since A is α-closed, $bcl(A) \subseteq \alpha cl(A) = A \subseteq U$. Therefore $bcl(A) \subseteq U$. Hence A is γgb-closed. ■

The converse of the above theorem need not be true as seen from the following example.

Example 3.11 From Example 3.7, let $A = \{b, c\}$. Here A is a γgb-closed set but not an α-closed set of (X, τ).

Theorem 3.12 Every semi-closed set is γgb-closed set.

Proof Let A be a semi-closed set in X such that $A \subseteq U$, where U is γ-open. Since A is semi-closed, $bcl(A) \subseteq scl(A) = A \subseteq U$. Therefore $bcl(A) \subseteq U$. Hence A is a γgb-closed set in X. ■

The converse of the above theorem need not be true as seen from the following example.

Example 3.13 Let $X = \{a, b, c\}$ with the topology $\tau = \{X, \phi, \{b, c\}\}$ and $\gamma(A) = A$ for all $A \in \tau$. Let $A = \{b\}$. Here A is a γgb-closed set but not a semi-closed set of (X, τ).

Theorem 3.14 Every preclosed set is γgb-closed set.

Proof Let A be a preclosed set in X such that $A \subseteq U$, where U is γ-open. Since A is preclosed, $bcl(A) \subseteq pcl(A) = A \subseteq U$. Therefore $bcl(A) \subseteq U$. Hence A is γgb-closed set in X. ■

The converse of the above theorem need not be true as seen from the following example.

Example 3.15 From Example 3.5, let $A = \{a, b\}$. Here A is a γgb-closed set but not a pre-closed set of (X, τ).

Theorem 3.16 Every g^*-closed set is γgb-closed set.

Proof Let A be a g^*-closed set in X such that $A \subseteq U$, where U is γ-open. Since every open set is g-open and A is g^*-closed, $bcl(A) \subseteq cl(A) \subseteq U$. Therefore $bcl(A) \subseteq U$. Hence A is a γgb-closed set in X. ■

The converse of the above theorem need not be true as seen from the following example.

Example 3.17 Let $X = \{a, b, c\}$ with the topology $\tau = \{X, \phi, \{b\}\}$ and $\gamma(A) = A$ for all $A \in \tau$. Let $A = \{a\}$. Here A is a γgb-closed set but not a g^*-closed set of (X, τ).

Theorem 3.18 Every gα-closed set is γgb-closed set.

Proof Let A be a gα-closed set in X such that $A \subseteq U$, where U is γ-open. Since every open set is α-open and A is gα-closed, $bcl(A) \subseteq \alpha cl(A) \subseteq U$. Therefore $bcl(A) \subseteq U$. Hence A is a γgb-closed set in X. ■

The converse of the above theorem need not be true as seen from the following example.

Example 3.19 From Example 3.5, let $A = \{a, b\}$. Here A is a γgb-closed set but not a gα-closed set of (X, τ).

Theorem 3.20 Every g-closed set is γgb-closed set.

Proof Let A be a g-closed set in X such that $A \subseteq U$, where U is γ-open. Since A is g-closed, $bcl(A) \subseteq cl(A) \subseteq U$. Hence A is γgb-closed. ■

The converse of the above theorem need not be true as seen from the following example.

Example 3.21 From Example 3.5, let $A = \{c\}$. Here A is a γgb-closed set but not a g-closed set of (X, τ).

Theorem 3.22 Every g^*b-closed set is γgb-closed set.

Proof Let A be a g^*b-closed set in X such that $A \subseteq U$, where U is γ-open. Since every open set is g-open and A is g^*b-closed, $bcl(A) \subseteq U$. Hence A is γgb-closed. ■

The converse of the above theorem need not be true as seen from the following example.

Example 3.23 Let $X = \{a, b, c\}$ with the topology $\tau = \{X, \phi, \{a\}\}$ and $\gamma(A) = X$ for all $A \in \tau$. Let $A = \{a, b\}$. Here A is a γgb-closed set but not a g^*b-closed set of (X, τ).

Theorem 3.24 Every αg-closed set is γgb-closed set.

Proof Let A be an αg-closed set in X such that $A \subseteq U$, where U is γ-open. Since every γ-open set is open and A is αg-closed, $bcl(A) \subseteq \alpha cl(A) \subseteq U$. Hence A is γgb-closed. ■

The converse of the above theorem need not be true as seen from the following example.

Example 3.25 Let $X = \{a, b, c\}$ with the topology $\tau = \{X, \phi, \{c\}\}$ and $\gamma(A) = X$ for all $A \in \tau$. Let $A = \{c\}$. Here A is a γgb-closed set but not an αg-closed set of (X, τ).

Theorem 3.26 Every gp-closed set is γgb-closed set.

Proof Let A be a gp-closed set in X such that $A \subseteq U$, where U is γ-open. Since every γ-open set is open and A is gp-closed, $bcl(A) \subseteq pcl(A) \subseteq U$. Hence A is γgb-closed. ■

The converse of the above theorem need not be true as seen from the following example.

Example 3.27 From Example 3.25, let $A = \{c\}$. Here A is a γgb-closed set but not a gp-closed set of (X, τ).

Theorem 3.28 Every gs-closed set is γgb-closed set.

Proof Let A be a gs-closed set in X such that $A \subseteq U$, where U is γ-open. Since every γ-open set is open and A is gs-closed, $bcl(A) \subseteq scl(A) \subseteq U$. Hence A is γgb-closed. ■

The converse of the above theorem need not be true as seen from the following example.

Example 3.29 From Example 3.25, let $A = \{c\}$. Here A is a γgb-closed set but not a gs-closed set of (X, τ).

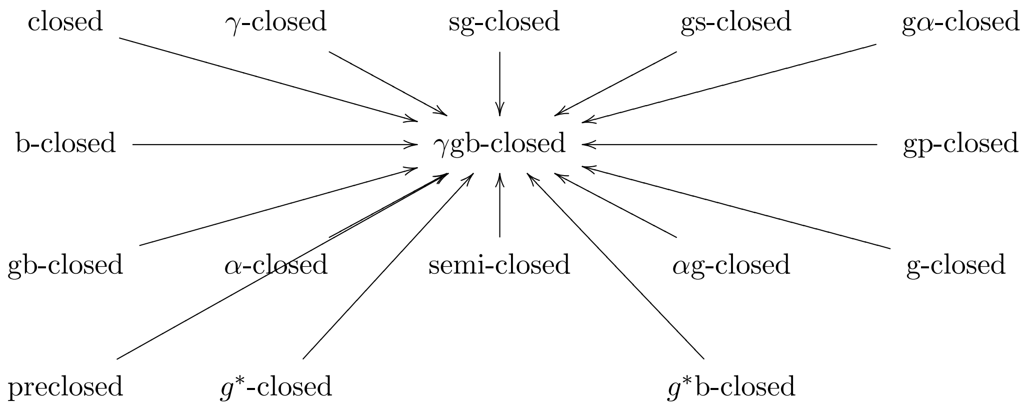
Theorem 3.30 Every sg-closed set is γgb-closed set.

Proof Let A be a sg-closed set in X such that $A \subseteq U$, where U is γ-open. Since every open set is semi-open and A is sg-closed, $bcl(A) \subseteq scl(A) \subseteq U$. Hence A is γgb-closed. ■

The converse of the above theorem need not be true as seen from the following example.

Example 3.31 From Example 3.25, let $A = \{c\}$. Here A is a γgb-closed set but not a sg-closed set of (X, τ).

Remark 3.32 We have the following implications but none of this implications are reversible.



Theorem 3.33 If X is a γ-regular space then every γgb-closed is gsp-closed.

Proof Let A be a γgb-closed set in X such that $A \subseteq U$, where U is open. Since every open set is γ-open and A is γgb-closed, $spcl(A) \subseteq bcl(A) \subseteq U$. Hence A is gsp-closed. ■

Theorem 3.34 If A is open and wg-closed then A is a γgb-closed.

Proof Let A be an open set and a wg-closed set in X such that $A \subseteq U$, where U is γ-open. Since every γ-open set is open and A is wg-closed, $bcl(A) \subseteq cl(A) = cl(int(A)) \subseteq U$. Hence A is γgb-closed. ■

Theorem 3.35 If A is γ-open and γgb-closed then A is b-closed.

Proof Let A be γ-open and γgb-closed. As $A \subseteq A$, we have $bcl(A) \subseteq A$, also $A \subseteq bcl(A)$, therefore $bcl(A) = A$. Hence A is b-closed. ■

Theorem 3.36 The intersection of a γgb-closed set and a γ-closed set is always γgb-closed.

Proof Let A be γgb-closed and F be γ-closed. Assume that U is γ-open set such that $A \cap F \subseteq U$, set $G = X \setminus F$. Then $A \subseteq U \cup G$, since G is γ-open, then $U \cup G$ is γ-open and since A is γgb-closed, then

$bcl(A) \subseteq U \cup G$. Now, $bcl(A \cap F) \subseteq bcl(A) \cap bcl(F) = bcl(A) \cap F \subseteq (U \cup G) \cap F = (U \cap F) \cup (G \cap F) = (U \cap F) \cup \phi \subseteq U$. Hence $A \cap F$ is γgb-closed. ■

The union of two γgb-closed sets need not be γgb-closed in general. It is shown by the following example.

Example 3.37 Let $X = \{a, b, c\}$ with the topology $\tau = \{X, \phi, \{a, b\}\}$ and $\gamma(A) = A$ for all $A \in \tau$. Let $A = \{a\}$ and $B = \{b\}$. Here A and B are γgb-closed but $A \cup B = \{a, b\}$ is not γgb-closed, since $\{a, b\}$ is γ-open and $bcl(\{a, b\}) = X$.

Definition 3.38 [9] Let A be a subset of a space X. A point $x \in X$ is said to be a b-limit point of A if for each b-open set U containing x, we have $U \cap (A \setminus \{x\}) \neq \phi$. The set of all b-limit points of A is called the b-derived set of A and is denoted by $D_b(A)$.

Corollary 3.1 [9] If $D(A) \subseteq D_b(A)$ for every subset A of X. Then for any subsets F and B of X, we have $bCl(F \cup B) = bCl(F) \cup bCl(B)$.

Theorem 3.39 If $D(A) \subseteq D_b(A)$ for every subset A of X. Then the finite union of γgb-closed sets is always a γgb-closed set.

Proof Let A and B be two γgb-closed sets, and let $A \cup B \subseteq U$, where U is γ-open. Since A and B are γgb-closed sets, therefore $bcl(A) \subseteq U$ and $bcl(B) \subseteq U$ implies $bcl(A) \cup bcl(B) \subseteq U$. So, by Corollary 3.1 we have $bcl(A) \cup bcl(B) = bcl(A \cup B)$. Therefore $bcl(A \cup B) \subseteq U$. Hence $A \cup B$ is a γgb-closed set. ■

The intersection of two γgb-closed sets need not be γgb-closed in general. It is shown by the following example.

Example 3.40 Let $X = \{a, b, c\}$ with the topology $\tau = \{X, \phi, \{c\}\}$ and $\gamma(A) = A$ for all $A \in \tau$. Let $A = \{a, c\}$ and $B = \{b, c\}$. Clearly, A and B are γgb-closed sets, since X is their only γ-open superset. But $C = \{c\} = A \cap B$ is not γgb-closed, since $C \subseteq \{c\} \in \tau_\gamma$ and $bcl(C) = X \not\subseteq \{c\}$.

Theorem 3.41 If a subset A of X is γgb-closed and $A \subseteq B \subseteq bcl(A)$, then B is a γgb-closed set in X.

Proof Let A be a γgb-closed set such that $A \subseteq B \subseteq bcl(A)$. Let U be a γ-open set of X such that $B \subseteq U$. Since A is γgb-closed, we have $bcl(A) \subseteq U$. Now $bcl(A) \subseteq bcl(B) \subseteq bcl[bcl(A)] = bcl(A) \subseteq U$. That is $bcl(B) \subseteq U$, where U is γ-open. Therefore B is a γgb-closed set in X. ■

The converse of the above theorm need not be true in general as seen from the following example.

Example 3.42 Let $X = \{a, b, c\}$ with the topology $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$ and $\gamma(A) = A$ for all $A \in \tau$. Let $A = \{b\}$ and $B = \{b, c\}$. Then A and B are γgb-closed sets in (X, τ). But $A \subseteq B \not\subseteq bcl(A)$.

Theorem 3.43 For each $x \in X$, $\{x\}$ is γ-closed or $X \setminus \{x\}$ is γgb-closed in X.

Proof Suppose that $\{x\}$ is not γ-closed, then $X \setminus \{x\}$ is not γ-open. Let U be any γ-open set such that $X \setminus \{x\} \subseteq U$, implies $U = X$. Therefore $bcl(X \setminus \{x\}) \subseteq U$. Hence $X \setminus \{x\}$ is γgb-closed. ■

Theorem 3.44 A subset A of X is γgb-closed if $bcl(\{x\}) \cap A \neq \phi$, holds for every $x \in bcl(A)$.

Proof Let U be a γ-open set such that $A \subseteq U$ and let $x \in bcl(A)$. By assumption, there exists a point $z \in bcl(\{x\})$ and $z \in A \subseteq U$. It follows that $U \cap \{x\} \neq \phi$, hence $x \in U$, this implies $bcl(A) \subseteq U$. Therefore A is γgb-closed. ■

Theorem 3.45 If a subset A of a space X is γgb-closed then $bcl(A) \setminus A$ does not contain any non-empty γ-closed set.

Proof Suppose that A is a γgb-closed set in X. We prove the result by contradiction. Let F be a γ-closed set such that $F \subseteq bcl(A) \setminus A$ and $F \neq \phi$. Then $F \subseteq X \setminus A$ which implies $A \subseteq X \setminus F$. Since A is γgb-closed and $X \setminus F$ is γ-open, therefore $bcl(A) \subseteq X \setminus F$, that is $F \subseteq X \setminus bcl(A)$. Hence

$F \subseteq bcl(A) \cap (X \setminus bcl(A)) = \phi$. This shows that, $F = \phi$ which is a contradiction. Hence $bcl(A) \setminus A$ does not contain any non-empty γ-closed set in X. ■

Theorem 3.46 A γgb-closed set A is b-closed if $bcl(A) \setminus A$ is γ-closed.

Proof Let $bcl(A) \setminus A$ be a γ-closed set and A be γgb-closed. Then by Theorm 3.45, $bcl(A) \setminus A$ does not contain any non-empty γ-closed subset. Since $bcl(A) \setminus A$ is γ-closed and $bcl(A) \setminus A = \phi$, this shows that A is b-closed. ■

Theorem 3.47 If A is γgb-closed and γ-closed then $bcl(A) \setminus A$ is γ-closed.

Proof If A is a γgb-closed set which is also γ-closed, then by Theorm 3.45, $bcl(A) \setminus A = \phi$, which is γ-closed. ■

References

[1] D. Vidhya and R. Parimelazhagan. g^*b-closed sets in topological spaces. *Int. J. Contemp. Math. Sciences*, 27:1305:1312, 2012.

[2] N. Levine. Generalized closed sets in topology. *Rend. Circ. Math. Palermo*, 19, 89:96 1970.

[3] J. Dontchev. On generalizing semi-preopen sets. *Mem. Fac. Sci. Kochi. Ser. A Math.*, 16:35:48, 1995.

[4] H. Maki, R. Devi, and K. Balachandran. Generalized α-closed sets in topology. *Bull. Fukuoka Univ. E. Part*, III(42):13:21, 1993.

[5] A. S. Mashhour, M. E. Abd EI-Monsef, and S. N. El-Deeb. On pre-continuous and weak pre-continuous mapping. *Proc. Math. Phys. Soc. Egypt*, 53:47:53, 1982.

[6] D. Andrijevic. Semi-preopen sets. *Mat. Vesink*, 38:24:32, 1986.

[7] N. Nagaveni. *Studies on generalisation of homeomorphisms in topological spaces.* Phd thesis, Bharathiar University, Coimbatore, 1999.

[8] D. Andrijevic. On b-open sets. *Mat. Vesink*, 48:59:64, 1996.

[9] A. A. Omari and M. S. M. Noorani. On generalized b-closed sets. *Bull. Malays. Math. Sci. Soc.*, 32(2):19:30, 2009.

[10] H. Ogata. Operation on topological spaces and associated topology. *Math. Japonica*, 36:175:184, 1991.

[11] N. Levine. Semi-open sets and semi-continuity in topological spaces. *Amer. Math. Monthly*, 70:36:41, 1963.

[12]

[13] H. Maki, R. Devi, and K. Balachandran. Associated topologies of generalized α-closed sets and α-generalized closed sets. *Mem. Fac. Sci. Kochi Univ. Ser. A Math.*, 15:51:63, 1994.

[14] H. Maki, J. Umehara, and T. Noiri. Every topological space is pre-$T\frac{1}{2}$. *Mem. Fac. Sci. Kochi Univ. Ser. A Math.*, 17:33:42, 1996.

[15] S. P. Arya and T. M. Nour. Characterizations of s-normal spaces. *Indian J. Pure Appl. Math.*, 21:717:719, 1990.

[16] P. Bhattacharyya and B. K. Lahiri. Semi generalized closed sets in topology. *Indian J. Math.*, 29:375:382, 1987.

[17] M. K. R. S. Veerakumar. Between closed sets and g-closed sets. *Mem. Fac. Sci. Kochi. Univ. Ser. A Math.*, 21:1:19, 2000.