Journal of Advanced Studies in Topology
eISSN: 2090-388X
pISSN: 2090-8288
Vol. 4, No. 1, 2013, 22–31

RESEARCH ARTICLE

Fuzzy Generalized Preregular Continuous Mappings in Fuzzy Topological Spaces

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(*Received: 3 August 2012, Accepted: 26 August 2012*)

In this paper the concepts of $fgpr$-continuous, $fgpr$-irresolute functions and $fgpr$-homeomorphism have been introduced and studied. we also introduce the concepts of $fgpr$-open and $fgpr$-closed mappings in fuzzy topological spaces.

Keywords: $fgpr$-continuous mappings; $fgpr$-irresolute functions; $fgpr$-open mappings; $fgpr$-closed mappings and $fgpr$-homeomorphism.

AMS Subject Classification: 54A40.

1. Introduction

Applying the concepts of g-closed sets in general topological spaces, several results in general topology were improved by introducing and studying g-closed maps by S. R. Malghan in 1982 and g-continuous maps by K. Balachandran, P. Sundaram and H. Maki in 1991. Further g^*-closed sets and g^*-continuous maps were introduced and studied by Veerekumar in the year 2000 for general topological spaces. The concepts of g^*-closed sets and $T^*\frac{1}{2}$, ${}^*T\frac{1}{2}$-spaces were introduced and studied by M. K. R. S. Veerakumar in the year 2000 for general topological spaces. Also g-continuous maps were introduced and studied. The concepts of fuzzy g^*-closed sets and fuzzy $T^*\frac{1}{2}$, ${}^*T\frac{1}{2}$ spaces were introduced and studied by M. S. Jayasheele Reddy in the year 2002 and R. Devi and M. Muthtamil Selvan in the year 2004, for fuzzy topological spaces. Also fuzzy g-continuous maps were introduced and studied.

In this paper, $fgpr$-continuous, $fgpr$-closed maps and related concepts in fts have been introduced and studied. It is observed that every f-continuous, $f\alpha$-continuous and f-pre continuous is a $fgpr$-continuous but not conversely. And also every fgp continuous function is a $fgpr$-continuous function but not conversely. A characterization of $fgpr$-continuous map is obtained. Results of composition of $fgpr$-continuous, $fgpr$-closed maps, and $fgpr$-homeomorphisms maps are obtained. It is observed that f-closed map is $fgpr$-closed map but not conversely. It is also observed that the image of $fgpr$-closed set is $fgpr$-closed under $fgpr$-irresolute closed map. Also the image of normal fts is normal under an f-continuous, $fgpr$-closed map.

Let X, Y and Z be sets. Throughout the present chapter $(X,\ \alpha)$, $(Y,\ \beta)$ and $(Z,\ \gamma)$ (or simply X, Y and Z) mean fuzzy topological spaces on which no separation axioms is assumed unless explicitely stated.

Before entering into our work we recall the following definitions, which are due to various authors.

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2. Preliminaries

If A is a subset of X with a topology τ, then the closure of A is denoted by τ-$cl(A)$ or $cl(A)$, the interior of A is denoted by τ-$int(A)$ or $int(A)$ and the complement of A in X is denoted by A^c.

Definition 2.1 A fuzzy set λ in a fts $(X,\ \tau)$ is called:

(1) a fuzzy pre-open set [1] if $\lambda \leq cl(int(\lambda))$ and a fuzzy pre-closed set if $cl(int(\lambda)) \leq \lambda$,
(2) a fuzzy α-open set [1] if $\lambda \leq int(cl(int(\lambda)))$ and a fuzzy α-closed set if $cl(int(cl(\lambda))) \leq \lambda$,
(3) fuzzy regular open set [2] if $int(cl\lambda) = \lambda$ and a fuzzy regular closed set if $cl(int(\lambda)) = \lambda$.

Definition 2.2 A fuzzy set λ in a fts $(X,\ \tau)$ is called:

(1) a fuzzy generalized closed set (briefly, fg-closed set)[1] if $cl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy open in X,
(2) a fuzzy generalized preregular closed set [3] (briefly $fgpr$-closed fuzzy set) if $pcl(\lambda) \subseteq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy regular open set in X.

Definition 2.3 Let X, Y be two fuzzy topolgogical spaces. A function $f : X \to Y$ is called:

(1) fuzzy continuous (briefly, f-continuous) [4] if $f^{-1}(\lambda)$ is fuzzy open set in X, for every fuzzy open set λ of Y,
(2) fuzzy α-continuous (briefly, $f\alpha$-continuous) [1] if $f^{-1}(\lambda)$ is fuzzy α-closed set in X, for every fuzzy closed set λ of Y,
(3) fuzzy precontinuous [1] if $f^{-1}(\lambda)$ is fuzzy pre-closed set in X, for every fuzzy closed set λ of Y,
(4) fuzzy gp-continuous (briefly, fgp-continuous) if $f^{-1}(\lambda)$ is fuzzy fgp-closed set in X, for every fuzzy closed set λ of Y,
(5) fuzzy generalized c-irresolute (briefly, fgc-irresolute) [4] if $f^{-1}(\lambda)$ is fuzzy g-closed set in X, for every fg-closed set λ of Y,
(6) fuzzy α generalized irresolute (briefly, $f\alpha g$-irresolute) [5] if $f^{-1}(\lambda)$ is fuzzy αg-closed set in X, for every $f\alpha g$-closed set λ of Y,
(7) fuzzy strongly continuous (briefly, fs-continuous) [4] if $f^{-1}(\lambda)$ is fuzzy open and fuzzy closed set in X, for every fuzzy set λ in Y,
(8) fuzzy perfectly continuous (briefly, fp-continuous) [4] if $f^{-1}(\lambda)$ is fuzzy open and fuzzy closed set in X, for every fuzzy open set λ in Y,
(9) fuzzy completely continuous (briefly, fc-continuous) [6] if $f^{-1}(\lambda)$ is fuzzy regular open set in X, for every fuzzy open set λ in Y.

Definition 2.4 Let X, Y be two fuzzy topological spaces. A map $f : X \to Y$ is called:

(1) fuzzy open (f-open) [7] if $f(\lambda)$ is fuzzy open set in Y, for every fuzzy open set of X,
(2) fuzzy g-open (briefly, fg-open) [4, 8] iff $f(\lambda)$ is fuzzy g-open set in Y, for every fuzzy open set in X.

Definition 2.5 A fuzzy topolgogical space (X, τ) is called a:

(1) fuzzy $T_{\frac{1}{2}}$ space [4] if every fg-closed set in X is a fuzzy closed set in X,
(2) fuzzy regular (briefly, f-regular) [9] if for each $x \in X$ and a fuzzy closed set λ with $\lambda(x) = 0$, there exist fuzzy open sets μ, γ such that $\mu(x) = 1$, $\lambda \leq \gamma$ and $\lambda \leq 1 - \gamma$.
(3) fuzzy normal (f-normal) [8] if for every fuzzy closed set λ and an fuzzy open set μ such that $\lambda \leq \mu$, there exists a fuzzy set γ such that $\lambda \leq \gamma^0 \leq \overline{\gamma} \leq \mu$.
(4) a fuzzy T_p^*-regular space [3] if every $fgpr$-closed set is a fuzzy preclosed set.
(5) a fuzzy ${}^*T_p$-regular space [3] if every fgp-closed set is $fgpr$-closed set.

3. Fuzzy Generalized Preregular Continuous Mappings

We introduce the following Definition:

Definition 3.1 Let X and Y be two fts. A function $f : X \to Y$ is said to be fuzzy generalized preregular continuous (briefly fgpr-continuous) if the inverse image of every fuzzy open set in Y is $fgpr$-open set in X.

Theorem 3.2 A function $f : X \to Y$ is $fgpr$-continuous iff the inverse image of every fuzzy closed set in Y is $fgpr$-closed set in X.

Proof Suppose the function $f : X \to Y$ is $fgpr$-continuous. Let λ be fuzzy closed set in Y. Then $1 - \lambda$ is fuzzy open set in Y. Since f is $fgpr$-continuous, $f^{-1}(1 - \lambda)$ is $fgpr$-open set in X. But $f^{-1}(1 - \lambda) = 1 - f^{-1}(\lambda)$ and so $f^{-1}(\lambda$ is $fgpr$-closed set in X.

Conversely, assume that the inverse image of every fuzzy closed set in Y is $fgpr$-closed set in X. Let μ be fuzzy open set in Y. Then $1-\mu$ is fuzzy closed set in Y. By hypothesis, $f^{-1}(1-\mu)$ is $fgpr$-closed set in X. But $f^{-1}(1 - \mu) = 1 - f^{-1}(\mu)$ and so $f^{-1}(\mu)$ is $fgpr$-open set in X. Hence f is $fgpr$-continuous. ■

Theorem 3.3 Every f-continuous function is $fgpr$-continuous.

Proof Let $f : X \to Y$ is f-continuous. Let λ be fuzzy closed set in Y. Since f is f-continuous, $f^{-1}(\lambda)$ is fuzzy closed set in X. And so, $f^{-1}(\lambda)$ is $fgpr$-closed set in X. Therefore f is $fgpr$-continuous. ■

The converse of the above theorem need not be true as seen from the following example.

Example 3.4 Let $X = Y = \{a,\ b,\ c\}$ and the fuzzy sets λ, μ and γ be defined as follows:

$$\lambda = \frac{0.4}{a} + \frac{0.5}{b} + \frac{0.7}{c},$$

$$\mu = \frac{1}{a} + \frac{0.9}{b} + \frac{0.8}{c},$$

$$\gamma = \frac{0}{a} + \frac{0.1}{b} + \frac{0.2}{c}.$$

Consider $\tau = \{0,\ 1,\ \lambda\}$ and $\sigma = \{0,\ 1,\ \mu\}$. Then $(X,\ \tau)$ and $(Y,\ \sigma)$ are fts. Define $f : X \to Y$ by $f(a) = a,\ f(b) = b$ and $f(c) = c$. Then f is $fgpr$-continuous but not f-continuous as the fuzzy set γ is fuzzy closed in Y and $f^{-1}(\gamma) = \gamma$ is not fuzzy closed set in X but $fgpr$-closed set in X. Hence f is $fgpr$-continuous.

Theorem 3.5

(1) Every fuzzy pre continuous function is $fgpr$-continuous.
(2) Every fuzzy α-continuous function is $fgpr$-continuous.

Proof Straightforward and follows from the Definitions. ■

The converse of the above theorem need not be true as seen from the following example.

Example 3.6 In the Example 3.4, f is $fgpr$-continuous but not f-pre continuous and f-α-continuous as the fuzzy set γ is fuzzy closed set in Y and $f^{-1}(\gamma) = \gamma$ is not fuzzy pre-closed set and not fuzzy α-closed in X but $fgpr$-closed set in X. Hence f is $fgpr$-continuous.

Theorem 3.7 Every fgp-continuous function is $fgpr$-continuous.

Proof Let $f : X \to Y$ is fgp-continuous. Let λ be fuzzy closed set in Y. Since f is fgp-continuous, $f^{-1}(\lambda)$ is fgp-closed set in X. Therefore $f^{-1}(\lambda)$ is $fgpr$-closed set in X as every fgp-closed set is $fgpr$-closed set. Hence f is $fgpr$-continuous. ■

The converse of the above theorem need not be true as seen from the following example.

Example 3.8 Let $X = Y = \{a,\ b,\ c\}$ and the fuzzy sets λ, μ and γ be defined as follows:

$$\lambda = \frac{0.4}{a} + \frac{0.5}{b} + \frac{0.7}{c},$$

$$\mu = \frac{0.6}{a} + \frac{1}{b} + \frac{1}{c},$$

and

$$\gamma = \frac{0.4}{a} + \frac{0}{b} + \frac{0}{c}.$$

Consider $\tau = \{0,\ 1,\ \lambda,\ \mu\}$ and $\sigma = \{0,\ 1,\ \mu\}$. Then $(X,\ \tau)$ and $(Y,\ \sigma)$ are fts. Define $f : X \to Y$ by $f(a) = a,\ f(b) = b$ and $f(c) = c$. Then f is $fgpr$-continuous but not fgp-continuous as the inverse image of fuzzy closed set γ in Y is $f^{-1}(\gamma) = \gamma$ which is not fgp-closed set in X.

Theorem 3.9 If a function $f : X \to Y$ is $fgpr$-continuous and X is fuzzy T_p^*-regular space, then f is f-precontinuous function.

Proof Let $f : X \to Y$ is $fgpr$-continuous. Let λ be fuzzy closed set in Y. Then $f^{-1}(\lambda)$ is $fgpr$-closed set in X, since f is $fgpr$-continuous. Also since X is fuzzy T_p^*-regular space, $f^{-1}(\lambda)$ is fuzzy pre-closed set in X. Hence f is f-precontinuous function. ■

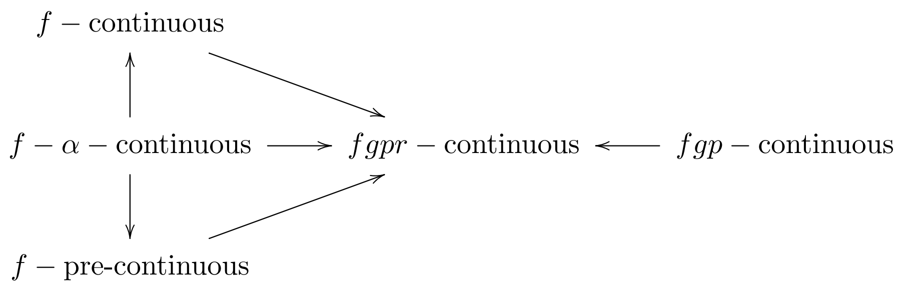
Theorem 3.10 If a function $f : X \to Y$ is fgp-continuous and X is fuzzy ${}^*T_p$-regular space, then f is $fgpr$-continuous function.

Proof Let $f : X \to Y$ is fgp-continuous. Let λ be fuzzy closed set in Y. Then $f^{-1}(\lambda)$ is fgp-closed set in X. Since X is fuzzy ${}^*T_p$-regular space, $f^{-1}(\lambda)$ is $fgpr$-closed set in X. Hence f is $fgpr$-continuous function. ■

Theorem 3.11 If $f : X \to Y$ is $fgpr$-continuous and $g : Y \to Z$ is f-continuous, then $g \circ f : X \to Z$ is $fgpr$-continuous function.

Proof Let λ be fuzzy closed set in Z. Then $g^{-1}(\lambda)$ is fuzzy closed set in Y, since g is f-continuous. And then $f^{-1}(g^{-1}(\lambda))$ is $fgpr$-closed set in X as f is $fgpr$-continuous. Now $(g \circ f)^{-1}(\lambda) = f^{-1}(g^{-1}(\lambda))$ is $fgpr$-closed set in X. Hence $g \circ f; X \to Z$ is $fgpr$-continuous function. ■

Remark 3.12 The following diagram shows the relationships of $fgpr$-continuous maps with some other fuzzy maps:



where $A \to B$ represents A implies B but not conversely.

Theorem 3.13 Let X, X_1 and X_2 be fts and $P_i : X_1 \times X_2 \to X_i (i = 1, 2)$ be the projection mappings. If $f : X \to X_1 \times X_2$ is $fgpr$-continuous then the $P_i \circ f : X \to X_i (i = 1, 2)$ is $fgpr$-continuous function.

Proof Let λ be fuzzy open set in $X_i (i = 1, 2)$, then $P_i^{-1}(\lambda)(i = 1, 2)$ is open in $X_1 \times X_2$ as the projection mapping P_i is f-continuous [10]. Since f is $fgpr$-continuous, $f^{-1}(P_i^{-1}(\lambda)) = (P_i \circ f)^{-1}(\lambda)(i = 1, 2)$ is $fgpr$-open in X. Hence $P_i \circ f$ is $fgpr$-continuous function. ■

Theorem 3.14 Every f-strongly continuous function is $fgpr$-continuous.

Proof Let $f : X \to Y$ be f-strongly continuous. Let λ be an fuzzy set in Y. Then $f^{-1}(\lambda)$ is both fuzzy open and fuzzy closed set in X. Therefore $f^{-1}(\lambda)$ is $fgpr$-fuzzy set in X. Hence f is $fgpr$-continuous. ■

The converse of the above theorem need not be true as seen from the following example.

Example 3.15 In the Example 3.4, f is $fgpr$-continuous but not f-strongly continuous for the fuzzy set γ in Y $f^{-1}(\gamma) = \gamma$ is not both fuzzy open and fuzzy closed set in X.

Theorem 3.16 Every f-perfectly continuous function is $fgpr$-continuous.

Proof Let $f : X \to Y$ is f-perfectly continuous. Let λ be an open fuzzy set in Y. Then $f^{-1}(\lambda)$ is both fuzzy open and fuzzy closed set in X. Therefore $f^{-1}(\lambda)$ is $fgpr$-open fuzzy set in X. Hence f is $fgpr$-continuous. ■

The converse of the above theorem need not be true as seen from the following example.

Example 3.17 In the Example 3.4, the function f is $fgpr$-continuous but not f-perfectly continuous for the fuzzy set γ in Y $f^{-1}(\gamma) = \gamma$ is not both fuzzy open and fuzzy closed set in X.

Theorem 3.18 Every f-completely continuous function is $fgpr$-continuous.

Proof Let $f : X \to Y$ be f-completely continuous. Let λ be an fuzzy open set in Y. Then $f^{-1}(\lambda)$ is fuzzy regular open set in X. Therefore $f^{-1}(\lambda)$ is fuzzy open set in X and therefore $f^{-1}(\lambda)$ is a $fgpr$-open set in X. Hence f is $fgpr$-continuous. ■

The converse of the above theorem need not be true as seen from the following example.

Example 3.19 In the Example 3.4, the function f is $fgpr$-continuous but not f-completely continuous as the fuzzy set μ is open in Y and $f^{-1}(\mu) = \mu$ is not fuzzy regular open set in X.

We introduce the following

Definition 3.20 A function $f : X \to Y$ is said to be fuzzy generalized preregular irresolute (briefly $fgpr$-irresolute) if the inverse image of every $fgpr$-closed set in Y is $fgpr$-closed fuzzy set in X.

Theorem 3.21 A function $f : X \to Y$ is $fgpr$-irresolute iff the inverse image of every fuzzy $fgpr$-open set in Y is $fgpr$-open set in X.

Proof Suppose the function $f : X \to Y$ is $fgpr$-irresolute. Let λ be fuzzy gpr-open set in Y. Then $1 - \lambda$ is fuzzy gpr-closed set in Y. Since f is $fgpr$-irresolute function, $f^{-1}(1 - \lambda)$ is $fgpr$-closed set in X. But $f^{-1}(1 - \lambda) = 1 - f^{-1}(\lambda)$ and so $f^{-1}(\lambda)$ is $fgpr$-open set in X.

Conversely, assume that the inverse image of every $fgpr$-open set in Y is $fgpr$-open set in X. Let λ be fuzzy gpr-closed set in Y. Then $1 - \lambda$ is fuzzy gpr-open set in Y. By hypothesis, $f^{-1}(1 - \lambda)$ is $fgpr$-open set in X. Now $f^{-1}(1 - \lambda) = 1 - f^{-1}(\lambda)$ and so $f^{-1}(\lambda)$ is $fgpr$-closed set in X. Hence f is $fgpr$-irresolute function. ■

Theorem 3.22 Every $fgpr$-irresolute function is $fgpr$-continuous.

Proof Let $f : X \to Y$ be $fgpr$-irresolute function. Let λ be a fuzzy closed set in Y. Then λ is $fgpr$-closed set in Y. Since f is $fgpr$-irresolute, $f^{-1}(\lambda)$ is $fgpr$-closed set in X. Hence f is $fgpr$-continuous. ■

The converse of the above theorem need not be true as seen from the following example.

Example 3.23 Let $X = Y = \{a,\ b,\ c\}$ and the fuzzy sets λ, μ and γ be defined as follows:

$$\lambda = \frac{0.4}{a} + \frac{0.5}{b} + \frac{0.7}{c},$$

$$\mu = \frac{0.1}{a} + \frac{0.5}{b} + \frac{0.3}{c},$$

$$\gamma = \frac{1}{a} + \frac{0.9}{b} + \frac{0.8}{c}$$

and

$$\delta = \frac{0}{a} + \frac{0.1}{b} + \frac{0.2}{c}.$$

Consider $\tau = \{0,\ 1,\ \lambda,\ \mu\}$ and $\sigma = \{0,\ 1,\ \gamma\}$. Then $(X,\ \tau)$ and $(Y,\ \sigma)$ are fts. Define $f : X \to Y$ by $f(a) = a,\ f(b) = b$ and $f(c) = c$. Then f is $fgpr$-continuous but not $fgpr$-irresolute as the fuzzy closed set δ in Y is $f^{-1}(\delta) = \delta$ which is not $fgpr$-closed set in X.

Theorem 3.24 Let $f : X \to Y$, $g : Y \to Z$ be two functions. If f and g are $fgpr$-irresolute functions, then $g \circ f : X \to Z$ is $fgpr$-irresolute functions.

Proof Let λ be $fgpr$-closed set in Z. Then $g^{-1}(\lambda)$ is $fgpr$-closed set in Y, since g is $fgpr$-irresolute. Since f is $fgpr$-irresolute, $f^{-1}(g^{-1}(\lambda))$ is $fgpr$-closed set in X. That is $(g \circ f)^{-1}(\lambda) = f^{-1}(g^{-1}(\lambda))$ is $fgpr$-closed set in X. Hence $g \circ f : X \to Z$ is $fgpr$-irresolute function. ■

Theorem 3.25 If $f : X \to Y$, $g : Y \to Z$ be two functions. If f is $fgpr$-irresolute and g is $fgpr$-continuous, then $g \circ f : X \to Z$ is $fgpr$-continuous.

Proof Let λ be fuzzy closed set in Z. Then $g^{-1}(\lambda)$ is $fgpr$-closed set in Y, since g is $fgpr$-continuous. Since f is $fgpr$-irresolute, $f^{-1}(g^{-1}(\lambda)) = (g \circ f)^{-1}(\lambda)$ is $fgpr$-closed set in X. Hence $g \circ f : X \to Z$ is $fgpr$-continuous. ■

Theorem 3.26 Let $f : X \to Y$ be a fgc-irresolute and f a closed map. then $f(\lambda)$ is a $fgpr$-closed set in Y, for every $fgpr$-closed set λ of X.

Proof Let λ be a $fgpr$-closed set of X. Let μ be a fg-open set of Y such that $f(\lambda) \leq \mu$. Then $f^{-1}(\mu)$ is a fg-open set in X, as f is fgc-irresolute function. Since $\lambda \leq f^{-1}(\mu)$ and λ is $fgpr$-closed set of X, $pcl(\lambda) \leq f^{-1}(\mu)$. Then $f(pcl(\lambda)) \leq f(f^{-1}(\mu)) \leq \mu$, $f(pcl(\lambda)) = pcl(fpcl(\lambda))$, as f is f-closed map. This implies $pcl(f(\lambda)) \leq pcl(f(pcl(\lambda))) \leq f(pcl(\lambda)) \leq \mu$. That is $pcl(f(\lambda)) \leq \mu$. Thus $f(\lambda)$ is a $fgpr$-closed set in Y. ■

Definition 3.27 A function $f : X \to Y$ is said to be fuzzy generalized preregular open (briefly $fgpr$-open) if the image of every fuzzy open set in X is $fgpr$-open set in Y.

Definition 3.28 A function $f : X \to Y$ is said to be fuzzy generalized preregular closed (briefly $fgpr$-closed) if the image of every fuzzy closed set in X is $fgpr$-closed set in Y.

Theorem 3.29 Every f-open map is $fgpr$-open map.

Proof Let $f : X \to Y$ be a f-open map and so is an fg-open map. Let λ be an fuzzy open set in X and so is an fg-open set in X. Then $f(\lambda)$ is fuzzy open set in Y since f is fuzzy open map. Therefore $f(\lambda)$ is $fgpr$-open set in Y. Hence f is $fgpr$-open map. ■

The converse of the above theorem need not be true as seen from the following example.

Example 3.30 Let $X = Y = \{a,\ b,\ c\}$ and the fuzzy sets λ, μ and γ be defined as follows:

$$\lambda = \frac{0.4}{a} + \frac{0.5}{b} + \frac{0.7}{c},$$

$$\mu = \frac{1}{a} + \frac{0.9}{b} + \frac{0.8}{c}$$

,

$$\gamma = \frac{0.6}{a} + \frac{0.5}{b} + \frac{0.3}{c}.$$

Consider $\tau = \{0,\ 1,\ \lambda\}$ and $\sigma = \{0,\ 1,\ \mu\}$. Then (X, τ) and $(Y,\ \sigma)$ are fts. Define $f : X \to Y$ by $f(a) = a,\ f(b) = b$ and $f(c) = c$. Then f is $fgpr$-open map but not f-open map as the fuzzy set γ is fuzzy closed in X and the image $f(\gamma) = \gamma$ is not fuzzy closed set in Y but $fgpr$-closed set in Y. Hence f is $fgpr$-open map.

Theorem 3.31 If $f : X \to Y$ is $fgpr$-open map and Y is fuzzy T_p^*-regular space. Then f is a f-open map.

Proof Let $f : X \to Y$ is $fgpr$-open map. Let λ be fuzzy open set in X and so is an fg-open set in X. Then $f(\lambda)$ is $fgpr$-open in Y. Since Y is fuzzy T_p^*-space, $f(\lambda)$ is fuzzy open set in Y. Hence f is f-open map. ■

Theorem 3.32 If $f : X \to Y$ is fgp-open map and Y is fuzzy ${}^*T_p$-regular space. Then f is a $fgpr$-open map.

Proof Let $f : X \to Y$ is fgp-open map. Let λ be fuzzy open set in X. Then $f(\lambda)$ is fgp-open in Y. Since Y is fuzzy ${}^*T_p$-space, $f(\lambda)$ is $fgpr$-fuzzy open set in Y. Hence f is $fgpr$-open map. ■

Theorem 3.33 Every f-closed map is $fgpr$-closed map.

Proof Let $f : X \to Y$ is f-closed map. Let λ be fuzzy closed set in X. Then $f(\lambda)$ is fuzzy closed set in Y. Therefore $f(\lambda)$ is $fgpr$-closed set in Y. Hence f is $fgpr$-closed map. ■

The converse of the above theorem need not be true as seen from the following example.

Example 3.34 In the Example 3.46, the function f is $fgpr$-closed map but not f-closed map as the fuzzy set γ is fuzzy closed set in X and its image $f(\lambda) = \lambda$ is $fgpr$-closed set in Y but not fuzzy closed set in Y.

Theorem 3.35 If $f : X \to Y$ is $fgpr$-closed map and Y is fuzzy T_p^*-regular space. Then f is a f-closed map.

Proof Let $f : X \to Y$ is $fgpr$-closed map. Let λ be fuzzy closed set in X. Then $f(\lambda)$ is $fgpr$-closed in Y. Since f is fuzzy T_p^*-space, $f(\lambda)$ is fuzzy closed set in Y. Hence f is f-closed map. ■

Theorem 3.36 A map $f : X \to Y$ is $fgpr$-closed iff for each fuzzy set λ of Y and for each fuzzy open μ such that $f^{-1}(\lambda) \leq \mu$, there is a $fgpr$-open set γ of Y such that $\lambda \leq \gamma$ and $f^{-1}(\gamma) \leq \mu$.

Proof Suppose f is $fgpr$-closed map. Let λ be a fuzzy set of Y, and μ be an fuzzy open set of X, such that $f^{-1}(\lambda) \leq \mu$. Then $\gamma = Y - f(X - \mu)$ is a $fgpr$-open set in Y, such that $\lambda \leq \gamma$ and $f^{-1}(\gamma) \leq \mu$.

Conversely, suppose that α is a fuzzy closed set of X. Then $f^{-1}(Y - f(\alpha)) \leq X - \alpha$, and $X - \alpha$ is fuzzy open set. By hypothesis, there is a $fgpr$-open set γ of Y such that $Y - f(\alpha) \leq \gamma$ and $f^{-1}(\gamma) \leq X - \alpha$. Therefore $\alpha \leq X - f^{-1}(\gamma)$. Hence $Y - \gamma \leq f(\alpha) \leq f(X - f^{-1}(\gamma)) \leq Y - \gamma$. This implies $f(\alpha) = Y - \gamma$. Since $Y - \gamma$ is $fgpr$-closed set, $f(\gamma)$ is $fgpr$-closed set and thus f is a $fgpr$-closed map. ■

Theorem 3.37 If a map $f : X \to Y$ is fgc-irresolute, $fgpr$-closed, fgp-closed and λ is $fgpr$-closed set of X, then $f(\lambda)$ is $fgpr$-closed set in Y.

Proof Let $f(\lambda) \leq 0$, where 0 is g-open fuzzy set in Y. Since f is fgc-irresolute, $f^{-1}(0)$ is a fuzzy g-open set, such that $\lambda \leq f^{-1}(0)$. Hence $pcl(\lambda) \leq f^{-1}(0)$, since λ is $fgpr$-closed set. Since f is fgp-closed map, $f(pcl(\lambda))$ is $fgpr$-closed set and $f(pcl(\lambda)) \leq 0$, which implies $pcl(f(pcl(\lambda))) \leq 0$. Since $f(pcl(\lambda))$ is $fgpr$-closed set, that is $pcl(f(\lambda)) \leq pcl(f(pcl(\lambda))) \leq 0$, and so $pcl(f(\lambda)) \leq 0$. Hence $f(\lambda)$ is $fgpr$-closed set in Y. ■

Theorem 3.38 Let $f : X \to Y$ be f-continuous and fgp-closed map. If λ is fgp-closed set in X and Y is fuzzy $T_{\frac{1}{2}}$ space, then $f(\lambda)$ is $fgpr$-closed set in Y.

Proof Let $f(\lambda) \leq 0$, where 0 is fg-open set in Y. Then 0 is fuzzy open set in Y, since Y is fuzzy $T_{\frac{1}{2}}$ space. Since f is f-continuous, $f^{-1}(0)$ is a fuzzy open set and so fg-open set, such that $\lambda \leq f^{-1}(0)$. Hence $pcl(\lambda) \leq f^{-1}(0)$, since λ is $fgpr$-fuzzy closed set. Since λ is $fgpr$-closed map, $f(pcl(\lambda))$ is $fgpr$-closed set and $f(pcl(\lambda)) \leq 0$, which implies $pcl(f(pcl(\lambda))) \leq 0$. Since $f(pcl(\lambda))$ is $fgpr$-closed set, that is $pcl(f(\lambda)) \leq pcl(f(pcl(\lambda))) \leq 0$, and so $pcl(f(\lambda)) \leq 0$. Hence $f(\lambda)$ is $fgpr$-closed set in Y. ■

Theorem 3.39 If $f : X \to Y$ is f-closed map and $h : Y \to Z$ is fgp-closed maps, then $h \circ f : X \to Z$ is $fgpr$-closed map.

Proof Let λ be fuzzy closed set in X. Then $f(\lambda)$ is f-closed set in Y. Since h is $fgpr$-closed map, $h(f(\lambda))$ is $fgpr$-closed set in Z. That is $(h \circ f)(\lambda) = h(f(\lambda))$ is $fgpr$-closed set in Z. Hence $h \circ f : X \to Z$ is $fgpr$-closed map. ■

Theorem 3.40 If f is a f-continuous, $fgpr$-closed map from a f-normal space X onto a fts Y, then Y is f-normal.

Proof Let a, b be two fuzzy closed sets of Y such that $a \leq 1 - b$. Then $f^{-1}(a)$ and $f^{-1}(b)$ are fuzzy closed sets of X such that $f^{-1}(a) \leq 1 - f^{-1}(b)$. Since X is f-normal, there exist fuzzy open sets u, v in x such that $f^{-1}(a) \leq u$, $f^{-1}(b) \leq v$ and $u \leq 1 - v$. Since f is $fgpr$-closed map, by definition, there exists $fgpr$-open sets g, h in Y, such that $a \leq g$, $b \leq h$, $f^{-1}(g) \leq u$ and $f^{-1}(h) \leq v$. Since $u \leq 1 - v$, we have $pint(g)$, $pint(h)$ are fuzzy open sets, such that $g^0 \leq 1 - h^0$. Since g is $fgpr$-open map, a is fuzzy closed set and so is fg-closed set and $a \leq g$ implies $a \leq pint(g)$. Similarly $b \leq pint(h)$, and $g^0 \leq 1 - h^0$. Hence Y is f-normal. ■

Theorem 3.41 Let $f : X \to Y$ be an f-continuous, open and $fgpr$-closed surjection. If X is regular fts then Y is regular.

Proof Let $q \in Y$ and $p \in X$ such that $f(p) = q$. Let g be an fuzzy open set in Y such that $g(q) = 1$. Then $(f^{-1}(g))(p) = g(f(p)) = 1$ and $f^{-1}(g)$ is fuzzy open set in X. Since X is regular fts, there is an fuzzy open set h in X, such that $h(p) = 1$ and $h \leq \overline{h} \leq f^{-1}(g)$. Since f is f-open, $f(h)$ is an fuzzy open set such that $(f(h))(q) = 1$, and $f(h) \leq f(\overline{h}) \leq g$. Since f is $fgpr$-closed, $f(pcl(h))$ is $fgpr$-closed set such that $f(pcl(h)) \leq g$, g is fuzzy open set and so is fg-open set. It follows that $pcl(f(pcl(h))) \leq g$. And hence $f(h) \leq pcl(f(h)) \leq pcl(f(pcl(h))) \leq g$. That is $f(h) \leq pcl(f(h)) \leq g$. Hence Y is regular fts. ■

Theorem 3.42 Let $f : X \to Y$, $h : Y \to Z$ be two maps such that $h \circ f : X \to Z$ is $fgpr$-closed map.

(1) If f is f-continuous and surjective, then h is $fgpr$-closed map.
(2) If h is $fgpr$-irresolute and injective, then f is $fgpr$-closed map.

Proof (1) Let λ be a fuzzy closed set in Y. Then $f^{-1}(\lambda)$ is f-closed set in X. And so $h \circ f$ is $fgpr$-closed map, $(h \circ f)(f^{-1}(\lambda)) = h(\lambda)$ is $fgpr$-closed set in Z. Thus $h : Y \to Z$ is a $fgpr$-closed map.

(2) Let μ be a fuzzy closed set in Y. Then $(h \circ f)(\mu)$ is $fgpr$-closed set in Z, and so $h^{-1}(h \circ f)(\mu)$ is a $fgpr$-closed set in Y. Since h is injective, $f(\mu) = h^{-1}(h \circ f)(\mu)$ is $fgpr$-closed set in Y. Therefore f is a $fgpr$-closed map. ■

Definition 3.43 Let X and Y be two fts. A bijective map $f : X \to Y$ is called fuzzy-homeomorphism [11] (briefly f-homeomorphism) if f and f^{-1} are fuzzy continuous.

We introduce the following.

Definition 3.44 Let X and Y be two fts. A bijective map $f : X \to Y$ is called fuzzy generalized preregular homeomorphism (briefly $fgpr$-homeomorphism) if f and f^{-1} are $fgpr$-continuous.

Theorem 3.45 Every f-homeomorphism is $fgpr$-homeomorphism.

Proof Let $f : X \to Y$ be a f-homeomorphism. Then f and f^{-1} are f-continuous. Therefore f and f^{-1} are $fgpr$-continuous. Hence f is $fgpr$-homeomorphism. ■

The converse of the above theorem need not be true as seen from the following example.

Example 3.46 In the Example 3.4, the function f is $fgpr$-homeomorphism but not f-homeomorphism as the fuzzy set λ is open in X and its image $f(\lambda) = \lambda$ is not fuzzy open set in Y, $f^{-1} : Y \to X$ is not f-ccontinuous.

Theorem 3.47 Let $f : X \to Y$ be a bijective function. Then the following are equivalent:

(1) f is $fgpr$-homeomorphism.
(2) f is $fgpr$-continuous and $fgpr$-open maps.
(3) f is $fgpr$-continuous and $fgpr$-closed maps.

Proof (1) ⇒ (2): Let f be $fgpr$-homeomorphism. Then f and f^{-1} are $fgpr$-continuous. To prove that f is $fgpr$-open map. Let λ be an fuzzy open set in X. Then, since $f^{-1} : Y \to X$ is $fgpr$-continuous, $f(\lambda) = (f^{-1})^{-1}(\lambda)$ is $fgpr$-open in Y. Therefore $f(\lambda)$ is $fgpr$-open in Y. Hence f is $fgpr$-open map.

(2) ⇒ (1): Let f be $fgpr$-open and $fgpr$-continuous map. To prove that $f^{-1} : Y \to X$ is $fgpr$-continuous. Let λ be an fuzzy open set in X. Then $f(\lambda)$ is $fgpr$-open set in Y. Since f is $fgpr$-open map. Now $(f^{-1})^{-1}(\lambda) = f(\lambda)$ is $fgpr$-open set in Y. Therefore $f^{-1} : Y \to X$ is $fgpr$-continuous. Hence f is $fgpr$-homeomorphism.

(2) ⇒ (3): Let f be $fgpr$-continuous and $fgpr$-open map. To prove that f is $fgpr$-closed map. Let λ be a fuzzy closed set in X. Then $1 - \lambda$ is fuzzy open set in X. since f is $fgpr$-open map, $f(1 - \lambda)$ is $fgpr$-open in Y. Now $f(1-\lambda) = 1 - f(\lambda)$. Therefore $f(\lambda)$ is $fgpr$-closed in Y. Hence f is $fgpr$-closed map.

(3) ⇒ (4): Let f be $fgpr$-continuous and $fgpr$-closed map. To prove that f is $fgpr$-open map. Let λ be a fuzzy open set in X. Then $1 - \lambda$ is fuzzy closed set in X. Since f is $fgpr$-closed map, $f(1 - \lambda)$ is $fgpr$-closed in Y. Now $f(1 - \lambda) = 1 - f(\lambda)$. Therefore $f(\lambda)$ is $fgpr$-open in Y. Hence f is $fgpr$-open map. ■

Definition 3.48 Let X and Y be two fts. A bijective map $f : X \to Y$ is called fuzzy generalized preregular-c-homeomorphism (briefly, $fgpr$-c-homeomorphism) if f and f^{-1} are fuzzy gpr-irresolute.

Theorem 3.49 Let X, Y, Z be fuzzy topological spaces and $f : X \to Y$, $g : Y \to Z$ be $fgpr$-c-homeomorphisms then their composition $g \circ f : X \to Z$ is $fgpr$-c-homeomorphism.

Proof Let λ be $fgpr$-open set in Z. Then since $g : Y \to Z$ is $fgpr$-irresolute, $g^{-1}(\lambda)$ is $fgpr$-open set in Y. Also since $f : X \to Y$ $fgpr$-irresolute, $f^{-1}(g^{-1}(\lambda)) = (g \circ f)^{-1}(\lambda)$ is $fgpr$-open set in X. Therefore $g \circ f : X \to Z$ is $fgpr$-irresolute.

Again, let λ be a $fgpr$-open set in X. Then, since $f^{-1} : Y \to X$ is $fgpr$-irresolute, $(f^{-1})^{-1}(\lambda) = f(\lambda)$ is $fgpr$-open set in Y. Also $g^{-1} : Z \to Y$ is $fgpr$-irresolute, $(g^{-1})^{-1}(f(\lambda)) = g(f(\lambda)) = (g \circ f)(\lambda)$ is $fgpr$-open in Z. Therefore $(g \circ f)^{-1} : Z \to X$ is $fgpr$-irresolute. Hence $g \circ f$ is $fgpr$-c-homeomorphism. ■

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