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RESEARCH ARTICLE

Somewhat b-I-Continuous and Somewhat b-I-Open Functions in Ideal Topological Spaces

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In this paper, new classes of functions are introduced and studied by making use of b-I-open sets and b-I-closed sets. Relationship between the new classes and other classes of functions are established besides giving examples, counterexamples, properties and characterizations.

Keywords: b-I-open; b-I-closed; somewhat b-I-continuous; somewhat b-I-open; b-I-dense; b-I-separable.

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1. Introduction

Topological ideals have played an important role in topology for several years. It was the works of Newcomb [1], Rancin [2], Samuels [3] and Hamlet and Jankovic [4–8] which motivated the research in applying topological ideals to generalize the most basic properties in general topology. In 1992, Jankovic and Hamlett [7] introduced the notion of I-open sets in topological spaces. Abd El-Monsef et al. [9] investigated I-open sets and I-continuous functions. In 1996, Dontchev [10] introduced the notion of pre-I-open sets and obtained a decomposition of I-continuity. Quite recently, Hatir and Noiri [11] have introduced the notion of semi-I-open sets to obtain another new decomposition of continuity. In this paper, using the notion of b-I-open sets, the concepts of somewhat b-I-continuous functions and somewhat b-I-open functions are introduced and studied. Also characterizations for somewhat b-I-continuity is obtained besides giving examples and counterexamples.

Throughout this paper, $cl(A)$ and $int(A)$ denote the closure of A and the interior of A, respectively. Let (X, τ) be a topological space and I an ideal of subsets of X. An *ideal topological space*, denoted by (X, τ, I), is a topological space (X, τ) with an ideal I on X. For a subset $A \subset X$, $A^*(I) = \{x \in X : U \cap A \neq I$ for each neighborhood U of $x\}$ is called the local function [10] of A with respect to I and τ. We simply write A^* instead of $A^*(I)$ in case there is no chance for confusion. Recall that $Cl^*(A) = A \cup A^*$ defines a Kuratowski closure operator. In what follows the space (X, τ, I) is always taken to be an ideal topological space.

Definition 1.1 A subset A of an ideal space (X, τ, I) is said to be:

(1) I-open [9] if $A \subseteq int(A^*)$.
(2) semi-I-open [11] if $A \subseteq cl^*(int(A))$.
(3) b-I-open [12] if $A \subseteq int(cl^*(A)) \cup cl^*(int(A))$.

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Definition 1.2 A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be somewhat-I-continuous function if for $U \in \sigma$ and $f^{-1}(U) \neq \phi$ there exists an I-open set V in X such that $V \neq \phi$ and $V \subseteq f^{-1}(U)$.

Definition 1.3 A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be somewhat semi-I-continuous if for $U \in \sigma$ and $f^{-1}(U) \neq \phi$ there exists an semi I-open set V in X such that $V \neq \phi$ and $V \subseteq f^{-1}(U)$.

Definition 1.4 A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be I-continuous, the inverse image of each open set is I-open.

Definition 1.5 A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be semi-I-continuous [11], the inverse image of each open set is semi-I-open.

Definition 1.6 A function $f : (X, \tau) \to (Y, \sigma, I)$ is said to be I-open (resp. semi-I-open) function if the image of open set U in (X, τ) is I-open (resp. semi-I-open) in (Y, σ, τ).

Definition 1.7 A function $f : (X, \tau) \to (Y, \sigma, I)$ is said to be somewhat-I-open function provided that for $U \in \tau$ and $U \neq \phi$, there exists an open set V in Y such that $V \neq \phi$ and $V \subseteq f(U)$.

Definition 1.8 A function $f : (X, \tau) \to (Y, \sigma, I)$ is said to be somewhat semi-I open function provided that for $U \in \tau$ and $U \neq \phi$, there exists a semi-I-open set V in Y such that $V \neq \phi$ and $V \subseteq f(U)$.

2. Somewhat *b*-*I*-continuous functions

Definition 2.1 Let (X, τ, I) be ideal topological spaces and (Y, σ) be any topological space. A function $f : X \to Y$ is said to be somewhat b-I continuous function if for every $U \in \sigma$ and $f^{-1}(U) \neq \phi$ there exists a b-I-open set V in X such that $V \neq \phi$ and $V \subseteq f^{-1}(U)$.

Example 2.2 Let $X = \{a, b\}$, $\tau = \{\phi, X\}$, $I = \{\phi, \{a\}\}$, $Y = \{a, b\}$, $\sigma = \{\phi, X, \{a\}\}$. Now define a function $f : (X, \tau, I) \to (Y, \sigma)$ as follows: $f(a) = a$; $f(b) = a$. Then clearly f is somewhat b-I-continuous function.

Theorem 2.3 Every somewhat semi-I-continuous function is somewhat b-I-continuous function.

Proof Let $f : X \to Y$ be somewhat semi-I-continuous function. Let U be any open set in Y such that $f^{-1}(U) \neq \phi$. Since f is somewhat semi-I-continuous, there exists a semi-I-open set V in X such that $V \neq \phi$ and $V \subseteq f^{-1}(U)$. Since every semi-I-open set is b-I-open, there exist a b-I open set V such that $V \neq \phi$ and $V \subseteq f^{-1}(U)$, which implies that f is somewhat b-I-continuous function. ■

Remark 2.4 Converse of the above theorem need not be true in general which follows from the following example.

Example 2.5 Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b, c\}\}$, $I = \{\phi, \{a\}\}$, $Y = \{a, b, c\}$, $\sigma = \{\phi, X, \{a\}, \{c\}, \{a, c\}\}$. Define a function $f : (X, \tau, I) \to (Y, \sigma)$ by $f(a) = a$, $f(b) = b$, $f(c) = c$. Then f is somewhat b-I-continuous but not somewhat semi-I-continuous. Since the inverse image of $\{c\}$ in (Y, σ) is $\{c\}$ in (X, τ, I) which is not semi-I-open set.

Theorem 2.6 Every somewhat I-continuous function is somewhat semi-I-continuous function.

Theorem 2.7 Every somewhat I-continuous function is somewhat b-I-continuous function.

Proof Theorem follows from Theorem 2.3 and Theorem 2.6. ■

Remark 2.8 Converse of the above theorem need not be true in general which follows from the following example.

Example 2.9 In example 2.5, the function $f : X \to Y$ defined by $f(a) = a$, $f(b) = b$, $f(c) = c$ is somewhat b-I-continuous but not somewhat-I-continuous since the inverse image of $\{c\}$ is $\{c\}$ which is not I-open set.

Theorem 2.10 Let $f : (X, \tau, I) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ be any two functions. If f is somewhat b-I-continuous function and g is continuous function, then $g \circ f$ is somewhat b-I-continuous function.

Proof Let $U \in \eta$. Suppose that $g^{-1}(U) \neq \phi$. Since $U \in \eta$ and g is continuous function $g^{-1}(U) \in \sigma$. Suppose that $f^{-1}(g^{-1}(U)) \neq \phi$. Since by hypothesis f is somewhat b-I-continuous function, there exists a b-I-open set V in X such that $V \neq \phi$ and $V \subseteq f^{-1}(g^{-1}(U))$. But $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$, which implies that $V \subset (g \circ f)^{-1}(U)$. Then $g \circ f$ is somewhat b-I-continuous function. ■

Remark 2.11 In the above Theorem 2.10, if f is continuous function and g is somewhat b-I-continuous function, then it is not necessarily true that $g \circ f$ is somewhat b-I-continuous function. The following example serves this purpose.

Example 2.12 Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$, $I = \{\phi, \{a\}\}$, $\sigma = \{\phi, X, \{a\}, \{b, c\}\}$, $\eta = \{\phi, X, \{a\}, \{c\}, \{a, c\}\}$. Define a function $f : (X, \tau) \to (X, \sigma)$ by $f(a) = a$, $f(b) = b$, $f(c) = c$ and define $g : (X, \sigma) \to (X, \eta)$ by $g(a) = a$, $g(b) = b$, $g(c) = c$. Then clearly f is continuous function and g is somewhat b-I-continuous but $g \circ f$ is not a somewhat b-I-continuous function.

Definition 2.13 Let M be a subset of a topological space (X, τ). Then M is said to be b-I-dense in X if there is no proper b-I-closed set C in X such that $M \subset C \subset X$.

Theorem 2.14 Let $f : (X, \tau, I) \to (Y, \sigma)$ be a function. Then the following are equivalent:

(i) f is somewhat b-I-continuous function.
(ii) If C is a closed subset of Y such that $f^{-1}(C) \neq X$, then there is a proper b-I-closed subset D of X such that $D \supset f^{-1}(C)$.
(iii) If M is a b-I-dense subset of X then $f(M)$ is a dense subset of Y.

Proof $(i) \Rightarrow (ii)$: Let C be a closed subset of Y such that $f^{-1}(C) \neq X$. Then $Y - C$ is an open set in Y such that $f^{-1}(Y - C) = X - f^{-1}(C) \neq \phi$. By hypothesis (i) there exists a b-I-open set V in X such that $V \neq \phi$ and $V \subset f^{-1}(Y - C) = X - f^{-1}(C)$. This means that $X - V \supset f^{-1}(C)$ and $X - V = D$ is a b-I-closed set in X. This proves (ii).

$(ii) \Rightarrow (iii)$: Let M be a b-I-dense set in X. We have to show that $f(M)$ is dense in Y. Suppose not, then there exists a proper closed set C in Y such that $f(M) \subset C \subset Y$. Clearly $f^{-1}(C) \neq X$. Hence by (ii) there exists a proper b-I-closed set D such that $M \subset f^{-1}(C) \subset D \subset X$. This contradicts fact that M is b-I-dense in X.

$(iii) \Rightarrow (ii)$: Suppose that (ii) is not true. This means there exists a closed set C in Y such that $f^{-1}(C) \neq X$. But there is no proper b-I-closed set D in X such that $f^{-1}(C) \subseteq D$. This means that $f^{-1}(C)$ is b-I dense in X. But by (iii) $f(f^{-1}(C)) = C$ must be dense in Y, which is contradiction to the choice of C.

$(ii) \Rightarrow (i)$: Let $U \in \sigma$ and $f^{-1}(U) \neq \phi$. Then $Y - U$ is closed and $f^{-1}(Y - U) = X - f^{-1}(U) \neq \phi$. By hypothesis of (ii) there exists a proper b-I-closed set $D \supset f^{-1}(Y - U)$. This implies that $X - D \subset f^{-1}(U)$ and $X - D$ is b-I-open and $X - D \neq \phi$. ■

Theorem 2.15 Let (X, τ, I) be any ideal topological space. Let (Y, σ) be any topological space. A be an open set in X and $f : (A, \tau/A) \to (Y, \sigma)$ be somewhat b-I-continuous function such that $f(A)$ is dense in Y. Then any extension F of f is somewhat b-I-continuous function.

Proof Let U be any open set in (Y, σ, I^*) such that $F^{-1}(U) \neq \phi$. Since $f(A) \subset Y$ is dense in Y and $U \cap f(A) \neq \phi$ it follows that $F^{-1}(U) \cap A \neq \phi$. That is $f^{-1}(U) \cap A \neq \phi$. Hence by hypothesis on f, there exists a b-open set V in A such that $V \neq \phi$ and $V \subset f^{-1}(U) \subset F^{-1}(U)$ which implies F is somewhat b-I-continuous function. ■

Theorem 2.16 Let (X, τ, I) and (Y, σ, I^*) be any two ideal topological spaces, $X = A \cup B$ where A and B are open subsets of X and $f : (X, \tau, I) \to (Y, \sigma, I^*)$ be a function such that f/A and f/B are somewhat b-I-continuous function. Then f is somewhat b-I-continuous function.

Proof Let U be any open set in (Y, σ, I^*) such that $f^{-1}(U) \neq \phi$. Then $(f/A)^{-1}(U) \neq \phi$ or $(f/B)^{-1}(U) \neq \phi$ or both $(f/A)^{-1}(U) \neq \phi$ and $(f/B)^{-1}(U) \neq \phi$.

Case(i): Suppose $(f/A)^{-1}(U) \neq \phi$.

Since f/A is somewhat b-I-continuous, there exists a b-I-open set V in A such that $V \neq \phi$ and $V \subset (f/A)^{-1}(U) \subseteq f^{-1}(U)$. Since V is b-I-open in A and A is open in X, V is b-I-open in X. Thus f is somewhat b-I-continuous function.

Case(ii): Suppose $(f/B)^{-1}(U) \neq \phi$.

Since f/B is somewhat b-I-continuous, there exists a b-I-open set V in B such that $V \neq \phi$ and $V \subset (f/B)^{-1}(U) \subset f^{-1}(U)$. Since V is b-I-open in B and B is open in X, V is b-I-open in X. Thus f is somewhat b-I-continuous function.

Case(iii): Suppose $(f/A)^{-1}(U) \neq \phi$ and $(f/B)^{-1}(U) \neq \phi$.

This follows from both the cases (i) and (ii). Thus f is somewhat b-I-continuous function. ■

Definition 2.17 A topological space X is said to be b-I-separable if there exists a countable subset B of X which is b-I-dense in X.

Theorem 2.18 If f is somewhat b-I-continuous function from X on to Y and if X is b-I-separable, then Y is separable.

Proof Let $f : X \to Y$ be somewhat b-I-continuous function such that X is b-I-separable. Then by definition there exists a countable subset B of X which is b-I-dense in X. Then by Theorem 2.14, $f(B)$ is dense in Y. Since B is countable $f(B)$ is also countable which is dense in Y, which indicates that Y is separable. ■

3. *b-I*-weakly Equivalent Topologies

Definition 3.1 If X is a set and τ and σ are topologies for X, then τ is said to be weakly equivalent to σ provided if $U \in \tau$ and $U \neq \phi$, then there is an open set V in (X, τ) such that $V \neq \phi$ and $V \subset U$ and if $U \in \sigma$ and $U \neq \phi$, there is an open set V in (X, τ) such that $V \neq \phi$ and $V \subset U$.

Definition 3.2 If X is a set and τ and σ are topologies for X and I is ideal for X, then τ is said to be b-I-weakly equivalent to σ provided if $U \in \tau$ and $U \neq \phi$, then there is a b-I open set V in (X, σ) such that $V \neq \phi$ and $V \subset U$ and if $U \in \sigma$ and $U \neq \phi$, then there is a b-I-open set V in (X, τ, I) such that $V \neq \phi$ and $V \subset U$.

Theorem 3.3 Let $f : (X, \tau, I) \to (Y, \sigma)$ be somewhat continuous function and let τ^* be a topology for X, which is b-I-weakly eqivalent to τ then the function $f : (X, \tau^*, I) \to (Y, \sigma)$ is somewhat b-I-continuous.

Proof Let U be any open set in (Y, σ) such that $f^{-1}(U) \neq \phi$. Since by hypothesis $f : (X, \tau, I) \to (Y, \sigma)$ is somewhat continuous by definition there exists an open set O in (X, τ, I) such that $O \neq \phi$ and $O \subset f^{-1}(U)$. Since O is an open set in (X, τ, I) such that $O \neq \phi$ and since by hypothesis τ^* is b-I-weakly equivalent to τ by definition there exists a b-I-open set V in (X, τ^*) such that $V \neq \phi$ and $V \subset O \subset f^{-1}(U)$. So $O \subset f^{-1}(U)$. Thus for any open set U in (Y, σ) such that $f^{-1}(U) \neq \phi$, there exists a b-I-open set V in (X, τ^*, I) such that $V \subset f^{-1}(U)$. So $f : (X, \tau^*, I) \to (Y, \sigma)$ is somewhat b-I-continuous function. ■

Theorem 3.4 Let $f : (X, \tau, I) \to (Y, \sigma)$ be somewhat b-I-continuous function and let σ^* be a topology for Y, which is weakly equivalent to σ. Then the function $f : (X, \tau, I) \to (Y, \sigma^*)$ is somewhat b-I-continuous function.

Proof Let U be an open set in (Y, σ^*) such that $f^{-1}(U) \neq \phi$ which implies $U \neq \phi$. Since σ and σ^* are weakly equivalent there exists an open set W in (Y, σ) such that $W \neq \phi$ and $W \subset U$. Now, W is an open set such that $W \neq \phi$, which implies $f^{-1}(W) \neq \phi$. Now by hypothesis $f : (X, \tau) \to (Y, \sigma)$ is somewhat b-I continuous function. Therefore there exists b-I-open set V in X such that $V \subset f^{-1}(W)$. Now

$W \subset U$ implies $f^{-1}(W) \subset f^{-1}(U)$. So we have $V \subset f^{-1}(U)$, which implies that $f : (X, \tau, I) \to (Y, \sigma^*)$ is somewhat b-I-continuous function. ■

4. Somewhat *b-I*-open functions

Definition 4.1 A function $f : (X, \tau) \to (Y, \sigma, I)$ is said to be somewhat b-I-open function provided that for $U \in \tau$ and $U \neq \phi$ there exists a b-I-open set V in Y such that $V \neq \phi$ and $V \subseteq f(U)$.

Example 4.2 Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$, $I = \{\phi, \{a\}\}$, $\sigma = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$. Now define a function $f : (X, \tau) \to (Y, \sigma, I)$ as follows $f(a) = a$, $f(b) = b$ and $f(c) = c$. Then clearly f is somewhat b-I-open function.

Theorem 4.3 Every somewhat semi-I-open function is somewhat b-I-open function.

Proof Let $f : (X, \tau) \to (Y, \sigma, I)$ be a somewhat semi-I-open function. Let $U \in \tau$ and $U \neq \phi$. Since f is somewhat semi-I-open function there exists a semi-I-open set V in Y such that $V \neq \phi$ and $V \subset f(U)$. But every semi-I-open set is b-I-open. Therefore there exists a b-I-open set V in Y such that $V \neq \phi$ and $V \subset f(U)$, which implies that f is somewhat b-I-open function. ■

Remark 4.4 Converse of the above Theorem need not be true in general, which follows from the following example.

Example 4.5 Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{c\}, \{a, c\}\}$, $I = \{\phi, \{a\}\}$, $\sigma = \{X, \phi, \{a\}, \{b, c\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma, I)$ by $f(a) = a$, $f(b) = b$ and $f(c) = c$. Then f is somewhat b-I-open but not somewhat semi-I-open. Since the image of $\{c\}$ in (X, τ) is $\{c\}$ in (Y, σ, I) which is not semi-I-open set.

Theorem 4.6 Every somewhat I-open function is somewhat b-I-open function.

Proof Let $f : (X, \tau) \to (Y, \sigma, I)$ be somewhat I-open function. Let $U \in \tau$ and $U \neq \phi$. Since f is somewhat I-open function, there exists an open set $V \in \sigma$ such that $V \neq \phi$ and $V \subseteq f(U)$. But every I-open set is b-I-open. So there exists a b-I-open set $V \in \sigma$ such that $V \neq \phi$ and $V \subseteq f(U)$. Thus f is somewhat b-I-open function. ■

Remark 4.7 Converse of the above theorem need not be true in general, which follows from the following example.

Example 4.8 In example 4.5, the function $f : X \to Y$ defined by $f(a) = a$, $f(b) = b$, $f(c) = c$ is somewhat b-I-open but not somewhat I-open, since the image of $\{a, c\}$ in (X, τ) is $\{a, c\}$ in (Y, σ, I) which is not I-open set.

Theorem 4.9 If $f : (X, \tau) \to (Y, \sigma)$ is an open map and $g : (Y, \sigma) \to (Z, \eta, I)$ is somewhat b-I-open map then $g \circ f : (X, \tau) \to (Z, \eta, I)$ is somewhat b-I-open map.

Proof Let $U \in \tau$. Suppose that $U \neq \phi$. Since f is an open map $f(U)$ is open and $f(U) \neq \phi$. Thus $f(U) \in \sigma$ and $f(U) \neq \phi$. Since g is somewhat b-I-open map and $f(U) \in \sigma$ such that $f(U) \neq \phi$ there exists a b-I-open set $V \in \eta$, $V \subset g(f(U))$, which implies $g \circ f$ is somewhat b-I-open function. ■

Theorem 4.10 If $f : (X, \tau) \to (Y, \sigma, I)$ be one-one and onto mapping, then the following are equivalent:

(i) f is somewhat b-I-open map.
(ii) If C is a closed subset of X such that $f(C) \neq Y$, then there is a b-I-closed subset D of Y such that $D \neq Y$ and $D \supset f(C)$.

Proof $(i) \Rightarrow (ii)$: Let C be any closed subset of X such that $f(C) \neq Y$. Then $X - C$ is open in X and $X - C \neq \phi$. Since f is somewhat b-I-open, there exists a b-I-open set $V \neq \phi$ in Y such that $V \subset f(X - C)$. Put $D = Y - V$. Clearly D is b-I-closed in Y and we claim that $D \neq Y$. For if $D = Y$, then $V = \phi$ which is a contradiction. Since $V \subset f(X - C)$, $D = Y - V \supset Y - [f(X - C)] = f(C)$.

$(ii) \Rightarrow (i)$: Let U be any non empty open set in X. Put $C = X - U$. Then C is a closed subset of X and $f(X - U) = f(C) = Y - f(U)$ implies $f(C) \neq \phi$. Therefore, by (ii) there is a b-I-closed subset D of Y such that $D \neq Y$ and $f(C) \subset D$. Put $V = X - D$. Clearly V is a b-I-open set and $V \neq \phi$. Further, $V = X - D \subset Y - f(C) = Y - [Y - f(U)] = f(U)$. ■

Theorem 4.11 Let $f : (X, \tau) \to (Y, \sigma, I)$ be somewhat b-I-open function and A be any open subset of X. Then $f/A : (A, \tau/A) \to (Y, \sigma, I)$ be also somewhat b-I-open function.

Proof Let $U \in \tau/A$ such that $U \neq \phi$. Since U is open in A and A is open in (X, τ), U is open in (X, τ) and since by hypothesis $f : (X, \tau) \to (Y, \sigma, I)$ is somewhat b-I-open function, there exists a b-I-open set V in Y such that $V \subset f(U)$. Thus for any open set U in $(A, \tau/A)$ with $U \neq \phi$, there exists a b-open set V in Y such that $V \subset f(U)$ which implies f/A is somewhat b-I-open function. ■

Theorem 4.12 Let (X, τ) be topological space and (Y, σ, I) be a ideal topological space and $X = A \cup B$ where A and B are open subsets of X and $f : (X, \tau) \to (Y, \sigma, I)$ be a function such that f/A and f/B is somewhat b-I-open, then f is also somewhat b-I-open function.

Proof Let U be any open subset of (X, τ) such that $U \neq \phi$. Since $X = A \cup B$ either $A \cap U \neq \phi$ (or) $B \cap U \neq \phi$ (or) both $A \cap U \neq \phi$ and $B \cap U \neq \phi$. ■

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