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RESEARCH ARTICLE

Topological Concepts Applied to Image Segmentation

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This paper presents an automatic method relevant to medical image segmentation of magnetic resonance image with tumors, by means of fuzzy hybrid filter and connected component extraction. A fuzzy hybrid filtering technique (FH_3F) is used to denoise the medical image and the topological concepts are applied to extract connected components and edge detection.

Keywords: Image segmentation; Topological space; medical image; hybrid filter.

AMS Subject Classification: 68U10, 94D05.

1. Introduction

The most commonly used medical images are X-ray, CT scan and MR images. The images collected by different type of sensors are generally tainted by different types of noises. The disturbance of an image which interfere with the interpretation, will be denoted by image noise. There are several types of image "noise" that can interfere with the interpretation of an image. Denoising is a central pre-processing step for analysis of an image leading to medical imagery measurements. Several good reviews of noise removal technique can be found in the literature [1].

In medical image processing, the automated recognition of the meaningful image components and other regions of interest is a fundamental task commonly referred to as image segmentation. Existing methods of segmentation were summarized by Nikhil R. et al. [2]. Fu and Mui [3] categorized segmentation techniques into three classes namely, characteristic feature thresholding or clustering, edge detection and region extraction. Further viewing edge detection as a functional approximation problem has been considered by Hueckel [4]. We note that several good reviews of segmentation techniques can be found in the literature Bezdek et al. [5], Pham et al. [6], and Xu et al. [7]. Pastore et al. [8] have described an automatic method applicable to the segmentation of mediastinum computerized Axial Tomography(CAT) images with tumors, by means of Alternating Sequential Filters(ASFs) of Mathematical Morphology, and connected components extraction based on continuous topology concepts.

In all the above mentioned articles the concept of denoising has not been introduced. Denoising of an image provides a better noise removal of images. This paper provides a segmentation algorithm after the introduction of denoising concepts. In the following section, we provide the basic notions of topological spaces relevant to the edge detection.

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2. Basic Notions of Topological Space

This section presents basic definitions, preliminary of topological spaces and digital topological concepts, which will be used along the development of this article. Topology is a branch of mathematics that studies the properties of geometric figures that are preserved through deformations, twisting and stretching, without regard to size and absolute position. In topology, the important mathematical notions are those of continuity and of continuous transformations; tearing, which would generate discontinuities, is prohibited. We next provide a few fundamental definitions [9], useful for developing the required algorithm.

Definition 2.1 (Topological Space) A topology on a set X is a collection τ of subsets of X having the following properties:

(1) ϕ and $X \in \tau$.
(2) The union of the elements of any sub collection of τ is in τ.
(3) The intersection of the elements of any finite sub collection of τ is in τ.

The couplet (X, τ) is called as the topological space, and the members of τ are called open sets.

Definition 2.2 (Neighborhood of a point) A neighborhood of a point x in a topological space X is an open set U containing x.

Definition 2.3 (Metric and Distance function) A metric space is a non-empty set M together with a function $d : M \times M \to R$ satisfying the following conditions:

(1) $d(x, y) \geq 0$ for all $x, y \in M$.
(2) $d(x, y) = 0$ iff $x = y$.
(3) $d(x, y) = d(y, x)$ for all $x, y \in M$.
(4) $d(x, z) \leq d(x, y) + d(y, z)$ for all $x, y, z \in M$.

Then d is called a metric or distance function and $d(x, y)$ is called the distance between x and y.

If d_1 and d_2 are two metrics on M, then $d = d_1 + d_2$ is also a metric on M and if (M, d) is a metric space, then $d^1(x, y) = \frac{d(x,y)}{(1+d(x,y))}$ is also a metric on M. If a distance d is defined, it is possible to define the following: $B(x, r) = \{y \in M; d(x, y) < r\}$ an open ball with center x and radius r. By means of these, open sets can be generated: a set $A \subset M$ is open, if and only if, for every $x \in A$, there is an open ball with center x and radius r included in A. Then the collection made up of all open sets constitutes a topology termed as metric topology.

The following definitions [9] are basic in a topological space.

Definition 2.4 (Connected components) Given X, define an equivalence relation on X by setting $x \sim y$ if there is a connected subset of X containing both x and y. The equivalence classes are called the connected components of X.

Definition 2.5 Let (X, τ) be a topological space and $A \subseteq X$. Then the closure of A is defined as the intersection of all closed sets containing A and is denoted by $cl(A)$.

Definition 2.6 (Interior of A) Let (X, τ) be a topological space and $A \subseteq X$. Then the interior of A is defined as the union of all open sets contained in A and is denoted by $Int(A)$ or A^0.

Definition 2.7 (Boundary of A) Let (X, τ) be a topological space and $A \subseteq X$. Then the boundary of A is defined by the equation $Bd(A) = \overline{A} \cap \overline{(X - A)}$.

Definition 2.8 (Isolated Point) A point x of $A \subset X$ is isolated if there is a neighborhood $U \in U(x)$, such that $(U \backslash x) \cap A = \phi$.

The following are the notions of digital topology required for the present work.

In 1970 Rosenfeld [10] has suggested the concept of digital topology. Digital topology deals with the topological properties of digital images. It provides the theoretical foundation for important image processing operations such as connected component labeling and counting, border following, contour filling and thinning and their generalizations to three or higher dimensional images.

Digital topology is the theoretical basis for understanding certain properties of sets in images, i.e., it is mainly suited to black-white images or to segmented objects in images. The objective of digital topology is to identify, represent, measure, characterize, compare, index, simplify, localize, and visualize objects and components in images. Next we introduce the concept digital image.

Definition 2.9 (Digital image) A digital image [11] is a function $f : Z \times Z \to [0, 1, \ldots, N-1]$ in which $N-1$ is a positive whole number belonging to the natural interval $[1, 256]$. The functional value of 'f' at any point $p(x, y)$ is called the intensity or grey level of the image at that point and it is denoted by $f(p)$ and the digital image is called a grey level image.

The following metrics are defined on $Z \times Z$.

$$d_1(x, y) = \sum_{i=1}^{2} |x_i - y_i| \tag{1}$$

$$d_2(x, y) =^{max}_{1 \leq i \leq 2} |x_i - y_i| \tag{2}$$

$$d_3(x, y) = d_1(x, y) + d_2(x, y) \tag{3}$$

$$d(x, y) = \left(\frac{d_3(x, y)}{1 + d_3(x, y)} \right) \tag{4}$$

In our present work, we consider $Z \times Z$ with the metric topology induced by the metric d. We define a distance function similar to Geodesic distance function.

Definition 2.10 If $x, y \in X$ have different grey level values then, $D(x, y) = +\infty$. If $x, y \in X$ have the same grey level value, then $D(x, y) = d(x, y)$ where $d(x, y)$ is defined by equation (4). Then D is a metric.

The some neighborhoods of $x \in Z \times Z$ are defined as follows:

Definition 2.11 (4-neighbours of a point) The 4-neighbours [12] of a point $p(x, y)$ are its four horizontal and vertical neighbours $(x \pm 1, y)$ and $(x, y \pm 1)$. A point 'p' and its 4-neighbours is denoted by $N_4(p)$, i.e., $N_4(p) = \{q \in Z \times Z | d_1(p, q) \leq 1\}$.

Definition 2.12 (8-neighbours of a point) The 8-neighbours [12] of a point $p(x, y)$ consist of its 4-neighbours together with its four diagonal neighbours $(x+1, y \pm 1)$ and $(x-1, y \pm 1)$. A point 'p' and its 8-neighbours is denoted by $N_8(p)$, i.e., $N_8(p) = \{q \in Z \times Z / d_2(p, q) \leq 1\}$.

Definition 2.13 (Cross neighbours of a point) The cross neighbours [13] of a point $p(x, y)$ consists of the neighbours $(x + 1, y \pm 1)$ and $(x - 1, y \pm 1)$. A point 'p' and its cross neighbours is denoted by $C_4(p)$, i.e., $C_4(p) = N_8(p) - N_4(p)$.

Definition 2.14 (LT neighbours of a point) The LT neighbours [13] of a point $p(x, y)$ consists of the neighbours $(x - 1, y - 1)$ and $(x + 1, y + 1)$. A set consisting of 'p' and its LT neighbours is denoted by $L_3(p)$.

Definition 2.15 (RT neighbours of a point) The RT neighbours [13] of a point $p(x, y)$ consists of the neighbours $(x - 1, y + 1)$ and $(x + 1, y - 1)$. A set consisting of 'p' and its RT neighbours is denoted by $R_3(p)$.

Definition 2.16 (Grouping criteria) Let X be an grey level image and τ a topology associated to X: and let $Y \subset \tau$. Define $\varphi : X \times Y \to R$, such that $\varphi(x, A) = d(x, \rho(A))$, where $\rho(A)$ is a characterization of A (a collection of pixels having same grey level values as A) and d is a metric. Given a fixed ϵ and δ, let X be a grey level image and $A \in Y$, an element $x \in X$ is said to belong to A if $B(x, \epsilon) \cap A \backslash \{x\} \neq \phi$ and $\varphi(x, (A \backslash \{x\})) < \delta$. i.e., Y is a covering of X.

The next section provides the algorithm for the denoising of an image and segmentation.

Definition 2.17 (Fuzzy hybrid max filter (FH_3F)) In fuzzy hybrid max filter [14], the general 8-neighbour function is defined as:

$$F(p) = \begin{cases} 1, \text{ if } f(p) = hmv(p),\ p \in N_8(p); \\ 0, \text{ otherwise.} \end{cases} \tag{5}$$

where $hmv(p)$ is the hybrid max value, which is the maximum of median pixel value of LT neighbours of a point 'p', median pixel value of RT neighbours of a point 'p' and pixel value of 'p'.

3. Algorithm

In the first stage of the algorithm a fuzzy hybrid max filter (FH_3F) is used to denoise the medical images as in [7]. The advantage of this new type of filters is that they reduce random noise in the image leaving the original shape unaltered, thus resulting in less noise image.

In the second stage, the proposed algorithm employs the grouping criterion defined in the prior section to obtain the desired image segmentation. The algorithm starts by generating a set with an image point (the first pixel found at the beginning of the image), the order followed in scanning the image is from top to bottom, vertically and from left to right, horizontally. Based on this, it creates the image connected components according to the grouping criterion.

Algorithm to extract the connected components in an image: First we define the function ϕ, i.e, distance d and $\rho(A)$. Let d be the distance function $D(x, y)$ defined in the Definition 2.10 and $\rho(A)$ is the collection of points having same grey level values as pixels of A. Let ϵ be fixed, say $\epsilon = 0.7$.

Initially a set $A_n = \{x\}$ is created, such that $x \in X$ with $n = 1$. I is a set of indices.

Step 1: If there exists $y \in X$ such that $y \notin A_i$, for every $i = 1, 2, \ldots, n$ go to Step 2. If there is no $y \in X$ go to step 3.

Step 2: For $i = 1, 2, 3, \ldots, n$ if $B(x, 0.7) \cap A_i \backslash \{y\} \neq \phi$ and $\phi(x, A_i \backslash \{y\}) < \infty$, then $I = I \cup \{i\}$ and $A_i = A_i \cup \{y\}$ for every $i \in I$; otherwise $n = n + 1$; $A_n = A_n \cup \{y\}$.

Step 3: If $Y = \bigcup_{i \in I} A_i$ is obtained, the connected components are generated and Y is called covering.

Step 4: After connected components are obtained, the boundary and interior of each components are obtained.

The algorithms are programmed by using JAVA.

4. Experimental Result and Discussions

The proposed method is applied to a magnetic resonance image of a brain with tumor. First a fuzzy hybrid filtering technique (FH_3F) is implemented using MATLAB 7.0. Then the proposed algorithm is implemented and different connected components are marked in different colours. Finally, the correct tumor segmentation is obtained. The exact boundaries of the segmented tumor as well as its localization are calculated. From the segmentation obtained the tumor affected area is identified.

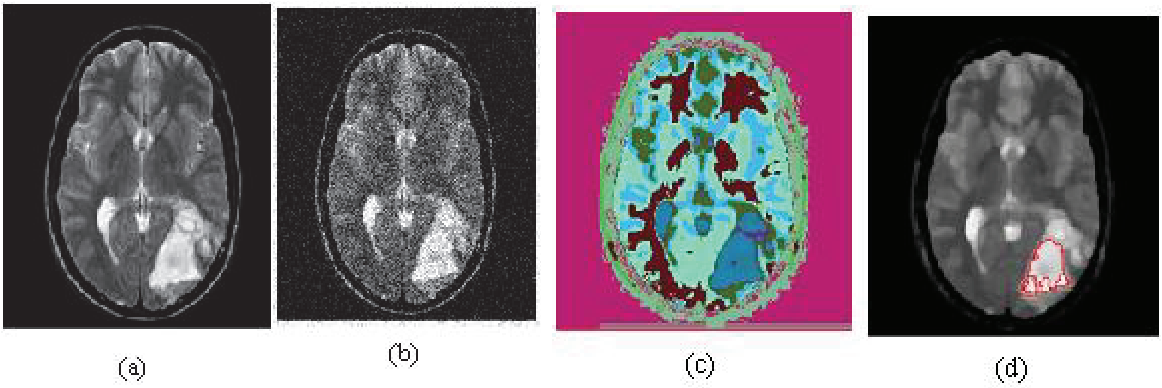

Figure 1. Brain Tumor image (a)Original image (b)the noisy MR image of brain with Gaussian noise of variance 0.0052 (c) Image with different connected components (d)Image resulting from segmentation and tumor identified.

5. Conclusion

In this work, a method applicable to the segmentation of medical images is introduced. To demonstrate the performance of the proposed techniques, the experiments have been conducted on MR brain tumor image. The potential of this method lies in the possibility of employing different distances and characterization of regions in the segmentation of other types of images. For instance when ϕ is defined by the Euclidean distance and $\rho(A)$ by a level of grey, the grouping algorithm is that known as image labeling.

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